PH3151 - Engineering Physics

UNIT - I

Mechanics

Multiparticle dynamics : Center of Mass (CM) - CM of continuous bodies - motion of the CM - Kinetic energy of system of Particles, Rotation of rigid bodies, Rotational Kinematics - rotational Kinetic energy and moment of Intertia - theorems of M.I - Moment of Inertia of continuous bodies - M.I of a diatomic molecule - torque - rotational dynamics of rigid bodies - conservation of angular momentum - rotational energy state of a rigid diatomic molecule - gyroscope - Torsional Pendulum - double Pendulum - Introduction to non-linear oscillations.

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(1) write a short notes on centre of Mass.

Definition

A point in the system at which mass of the body is supposed to be contentrated is called centre of mass of the body.

Examples.

i) Motion of Planets and its Satellite

- consider motion of the centre of mass of the earth and moon.
- Moon moves around the earth in a circular orbit.
- Earth moves round the sun in an elliptical orbit.
- Earth and moon has the mutual gravitational attractions

ii) Projectile Trajectory.

- when a cracker is fired at an angle with the horizontal it explodes in the air.
- Different pieces of the cracker follows different Parabolic Paths.

iii) Decay of a nucleus.

- spontaneous decay of radiactive nucleus into two fragments.
- obey the laws of conservation of energy and momentum.

centre of mass of two Point masses

i) when the masses are on Positive x-axis.

- The origin is taken arbitrarily
  - $m_1 m_2$ - masses
  - x_1, x_2 - Positions.

$$\boxed{x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 m_2}}$$

ii) when the origin coincides with any one of the masses.

$$x_{CM} = \frac{m_1(0) + m_2 x_2}{m_1 \cancel{+} m_2}$$

$$\boxed{x_{CM} = \frac{m_2 x_2}{m_1 + m_2}}$$

ii) when the origin coincides with the centre of mass itself

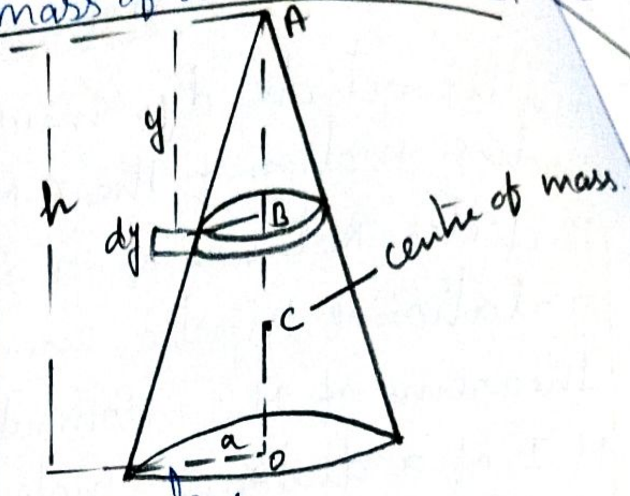
$$0 = \frac{m_1(-x_1) + m_2 x_2}{m_1 + m_2}$$

$$0 = m_1(\cancel{x}_1) + m_2 x_2$$

$$\boxed{m_1 x_1 = m_2 x_2}$$

The above equ is Known as Principle of moment.

② Derive an expressi... continous bodies of a cen... mass of a solid cone.



a - radius
h - height
ρ - density

If the solid cone is homogeneous its mass

$$m = \frac{1}{3}\pi a^2 h \cdot \rho$$

The centre of mass lies on the axis of symmetry AO

dy - thickness

Elementary disc of radius x, y - distance

The mass of elementary disc is

$$dm = \rho(\pi x^2)\, dy \text{ ——— (1)}$$

$$\frac{x}{a} = \frac{y}{h}$$

$$dm = \rho\pi\left(\frac{a}{h}\cdot y\right)^2 dy \text{ ——— (2)}$$

From equ (1)

$$y_{CM} = \frac{1}{m}\int y\, dm \text{ ——— (3)}$$

on substituting the value of dm we get.

y_{CM} ... (or) y_{CM} = ...

$$y_{CM} = \frac{1}{m}\int_0^h y \rho \pi \left(\frac{a}{h} y\right)^2 dy$$

(or)

$$y_{CM} = \frac{\rho \pi a^2}{m h^2}\int_0^h y^3 dy \quad \text{——— (4)}$$

limits $y = 0$ to $y = h$

$$y_{CM} = \frac{\rho \pi a^2}{m h^2}\left[\frac{y^4}{4}\right]_0^h$$

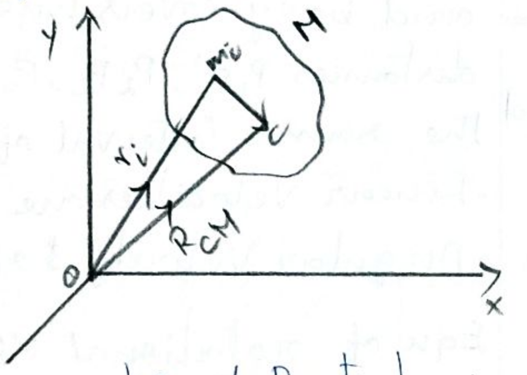
$$= \frac{\rho \pi a^2}{m h^2}\frac{h^4}{4} = \frac{\rho \pi a^2 h^2}{4m} \quad \text{——— (5)}$$

m - Total mass of the solid cone $= \frac{1}{3}\pi a^2 h . \rho$

$$Y_{CM} = \frac{\cancel{\rho}\cancel{\pi}\cancel{a^2} h^{\cancel{2}} . 3}{4 \cancel{\pi}\cancel{a^2} \cancel{h} . \cancel{\rho}}$$

$$Y_{CM} = \frac{3}{4} h$$

(3) <u>Derive an expression for kinetic energy of the system of Particles</u>



- Let n number of Particles in a system
- i^{th} particle of this system depends on the external force $\vec{F_i}$.
- Kinetic energy be

$$E_{Ki} = \frac{1}{2} m_i v_i^2$$

$$E_{Ki} = \frac{1}{2} m_i (v_i . v_i) \quad \text{——— (1)}$$

$\vec{r_i}$ - Position vector of the i^{th} Particle w.r.to O

$$\vec{r_i} = \vec{r_i'} + \vec{R}_{CM} \quad \text{——— (2)}$$

$\vec{R}_{CM}$ - Position vector of centre of mass w.r.to O Differentiating equ (2)

$$\frac{d\vec{r_i}}{dt} = \frac{d\vec{r_i'}}{dt} + \frac{d\vec{R}_{CM}}{dt} \quad \text{(or)}$$

$$v_i = v_i' + v_{CM} \quad \text{——— (3)}$$

Putting equ (3) in (1)

$$E_{Ki} = \frac{1}{2} m_i \left[(v_i' + v_{cm}) . (v_i' + v_{cm})\right]$$

$$= \frac{1}{2} m_i \left[(v_i'^2 + 2 v_i' + v_{cm} + v_{cm}^2)\right]$$

$$E_{Ki} = \frac{1}{2} m_i v_i'^2 + \frac{2}{2} m_i v_i' v_{CM} + \frac{1}{2} m_i v_{cm}^2$$

$$E_{Ki} = \frac{1}{2} m_i v_i'^2 + m_i v_i' v_{cm} + \frac{1}{2} m_i v_{cm}^2 \quad \text{——— (4)}$$

The sum of K.E of all the Particles can be obtained from (4)

$$E_K = \sum_{i=1}^{n} E_{Ki} = \sum_{i=1}^{n} \left(\frac{1}{2} m_i v_i'^2 + m_i v_i' . v_{cm} + \frac{1}{2} m_i v_{cm}^2\right]$$

$$E_K = \sum_{i=1}^{n} \frac{1}{2} m_i v_i'^2 + \sum_{i=1}^{n} m_i v_i' v_{cm} + \sum_{i=1}^{n} \frac{1}{2} m_i v_{cm}^2$$

$$E_K = \frac{1}{2} V_{CM}^2 \sum_{i=1}^{n} m_i + \sum_{i=1}^{n} \frac{1}{2} m_i v_i'^2 + V_{CM} \sum_{i=1}^{n} m_i v_i'$$

$$E_K = \frac{1}{2}V_{CM}^2 M + \sum_{i=1}^{n} \frac{1}{2} m_i v_i'^2 + V_{cm}\frac{d}{dt}\sum_{i=1}^{n} m_i \vec{v_i} \quad \text{—(5)}$$

Now last term in equ (5) is to zero

$$\sum_{i=1}^{n} m_i \vec{v_i} = 0$$

$$\sum_{i=1}^{n} m_i \vec{v_i} = \sum_{i=1}^{n} m_i(\vec{v_i} - R_{CM})$$

$$= \sum_{i=1}^{n} m_i \vec{v_i} - \sum_{i=1}^{m} m_i \vec{R}_{CM}$$

$$= MR_{CM} - MR_{CM} = 0$$

K.E of the system of Particles

$$E_K = \frac{1}{2} M V_{CM}^2 + \frac{1}{2}\sum_{i=1}^{n} m_i v_i'^2$$

$$= KK_{CN} + E_{K'} \quad \text{—— (6)}$$

where

$$E_{K_{CM}} = \frac{1}{2} V_{CM}^2 M$$

$$E'_K = \sum_{i=1}^{n} \frac{1}{2} m_i \vec{V_i'}^2 \quad \text{—— (7)}$$

(4) **Define rigid body. Derive and Explain Rotational Motion.**

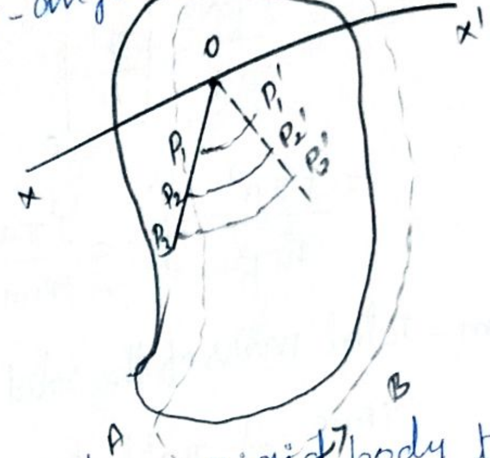
Rigid body.

A rigid body is defined as that body which does not undergo any change in shape or volume when external force are applied on it.

Rotational Motion.

when a body rotates a fixed axis its motion known as rotatory motion

- radius vector r
- θ - angular displacement



consider a rigid body that rotates about a fixed axis XOX' Passing through O and Perpendicular to the plane of Paper.

- Body rotate from the Position A to the Position B.
- Different Particles at P_1, P_2, P_3 and body covers unequal distances P_1P_1', P_2P_2', P_3P_3' .... in the same interval of time
- Linear velocities are different
- Angular Velocity is same.

Equ of rotational Motion

- Particle start rotating with angular velocity ω_0.
- angular acceleration α
- At any instant t, ω be the angular velocity of the Particle.

-angular displacement change in angular velocity in time

$t = \omega - \omega_0$

Angular acceleration

$= \frac{\text{change in angular velocity}}{\text{time taken.}}$

$$\alpha = \frac{\omega - \omega_0}{t} \quad \text{--- (1)}$$

$$\alpha t = \omega - \omega_0$$

$$\omega = \omega_0 + \alpha t \quad \text{--- (2)}$$

Average angular Velocity $= \left(\frac{\omega + \omega_0}{2}\right)$

Total angular displacement

= average angular velocity x time taken

$$\theta = \left(\frac{\omega + \omega_0}{2}\right) t \quad \text{--- (3)}$$

Substituting ω from the equ (2)

$$\theta = \left(\frac{\omega_0 + \alpha t + \omega_0}{2}\right) t = \left(\frac{2\omega_0 + \alpha t}{2}\right)$$

$$= \left(\frac{2\omega_0}{2} + \frac{\alpha t}{2}\right) t$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \quad \text{--- (4)}$$

From equ (1), $t = \left(\frac{\omega - \omega_0}{\alpha}\right)$ --- (5)

Using equ (5) in (3)

$$\theta = \left(\frac{\omega + \omega_0}{2}\right)\left(\frac{\omega - \omega_0}{\alpha}\right)$$

$$\theta = \left(\frac{\omega^2 - \omega_0^2}{2\alpha}\right)$$

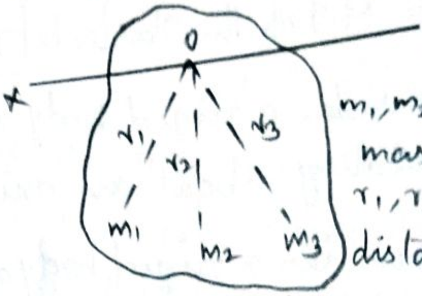
$$2\alpha\theta = \omega^2 - \omega_0^2 \quad \boxed{\omega^2 = \omega_0^2 + 2\alpha\theta} \text{--- (6)}$$

equ (2) (4) (6) are the rotational motion

(5) Derive and Discuss about rotational Kinetic energy and Moment of Inertia

i) Rotational Kinetic energy.

- Consider a rigid body a large number of Particles rotating about a fixed axis XOX'



m_1, m_2, m_3 masses

r_1, r_2, r_3 distances

K.E of the first Particle $= \frac{1}{2} m_1 v_1^2$

$= \frac{1}{2} m_1 (r_1 \omega^2)$

K.E of the second Particle

$= \frac{1}{2} m_2 r_2^2 \omega^2$

K.E of the third Particle

$= \frac{1}{2} m_3 r_3^2 \omega^2$

Sum of the K.E

$$E_K = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2$$

$$= \frac{1}{2}\omega^2 (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2)$$

$$E_K = \frac{1}{2}\omega^2 \Sigma m r^2$$

$$\boxed{I = \Sigma m r^2}$$

$$E_K = \frac{1}{2} I \omega^2$$

ii) Moment of Inertial or rotational motion.

The property of a body by virtue of which its opposes any change in its state of rotation about an axis is called the Moment of Inertia of it.

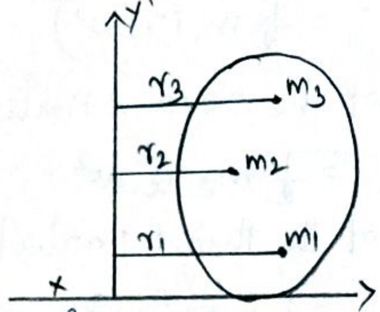
Moment of Inertia of a Particle

- m - mass of the Particle
- r - distance of the Particle from the axis of rotation

The M.I of the Particle $\boxed{I = mr^2}$

consider a rigid body of mass M, rotating about an axis xx'

consider a rigid body of Mass M, rotating about an axis xx'



$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \ldots\ldots$$

$$I = \Sigma m r^2$$

Angular Velocity $\omega = 1$ radian/sec

Rotational K.E $= E_R = \frac{1}{2} I \omega^2$

$$= \frac{1}{2} I \omega^2 = \frac{1}{2} I \cdot 1$$

$$E_R = \frac{1}{2} I \quad 2E_R = I$$

$$\boxed{I = 2E_R}$$

Significance

- Measure of Inertia for a given system in rotational motion.

⑥ Discuss the Theorems of Moment of Inertia.

- There are two important theorems

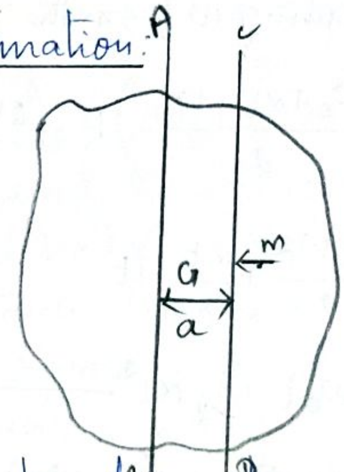
i) Parallel axis theorem

ii) Perpendicular axis theorem

i) Parallel axis theorem

The moment of Inertia of a rigid body about any axis is equal to the sum of its moment of inertia about a parallel axis through its centre of gravity and the product of the mass of the body and the square of the distance between the two Parallel axis.

Explanation:



G - Centre of gravity of a rigid body

M - Mass

AB Parallel to CD

I and I_G - Moment of Inertia

$$\boxed{I = I_G + Ma^2}$$

M.I of the whole body about CD

$$I_G = \Sigma m r^2$$

M.I of the axis AB

M.I

M.I of the Particle about the axis $AB = m(r+a)^2$

M.I about the whole body about AB

$$I = \Sigma m(r+a)^2$$

$$I = \Sigma m(r^2+a^2+2ar)$$

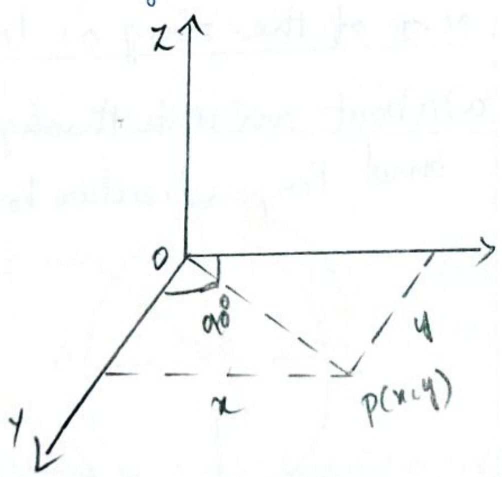
$$I = I_a + Ma^2 + 2a\Sigma mr$$

The body always balances about an axis through its centre of gravity. Σmr be zero

$$\boxed{I = I_G + Ma^2}$$

ii) Perpendicular axis theorem

It states that the moment of Inertia of a Plane lamina about an axis Perpendicular to its plane is equal to the sum of the moments of inertia of the Plane lamina about any two mutually Perpendicular axes in its own plane and intersecting each other at the Point where the Perpendicular axis Passes through it.



OX, OY - two mutually Perpendicular axes in the Plane of lamina, intersecting each other at the Point O.

OZ - Perpendicular to both OX and OY

I_x, I_y - Moments of Inertia of the Lamina about the axis OZ

$$\boxed{I_z = I_x + I_y}$$

Proof

- Consider the axes OX and OY

Moment of Inertia about $OX = my^2$

Moment of the entire lamina OX,

$$I_x = \Sigma my^2.$$

Moment of lamina about OY,

$$I_y = \Sigma mx^2$$

$$I_z = \Sigma mr^2 \quad \text{--- (1)}$$

$$r^2 = x^2+y^2 \quad \text{--- (2)}$$

Substituting equ (2) in equ (1)

$$I_z = \Sigma m(x^2+y^2)$$

$$I_z = \Sigma mx^2 + \Sigma my^2$$

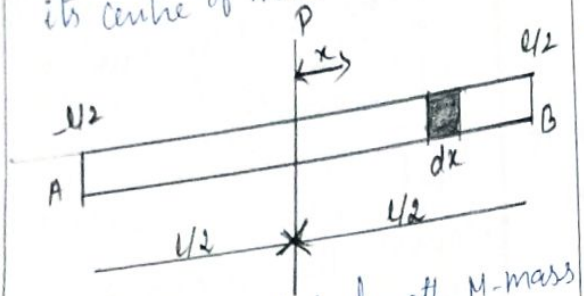
$$I_z = I_y + I_x$$

$$\boxed{I_z = I_x + I_y}$$

(7) Derive an expression for Moment of Inertia of continuous Bodies.

(1) M.I of a thin uniform rod

a) about an axis passing through its centre of mass and Perpendicular



AB-Uniform rod, l-length, M-mass

PQ-Perpendicular axis

Mass Per unit length of the rod

$$m = \frac{M}{l} \quad \text{(1)}$$

consider a Small element of length dx of the rod at a distance x from O

Mass of the element $= m.dx$

M.I of the element about the axis PQ

$$= \text{mass} \times (\text{distance})^2$$

$$= mdx.x^2$$

$$= mx^2 dx \quad \text{(2)}$$

Integrating with limits.

$x = -l/2$, $x = l/2$.

$$I = \int_{-l/2}^{l/2} mx^2\,dx = m\int_{-l/2}^{l/2} x^2 dx$$

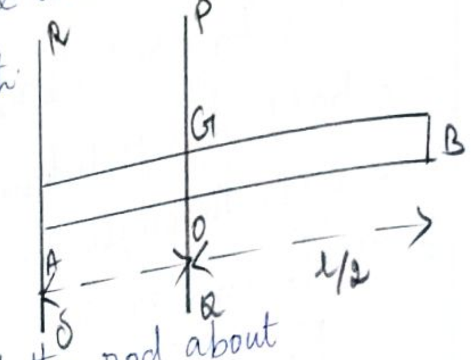
$$= m\left[\frac{x^3}{3}\right]_{-l/2}^{l/2} = m\left[\frac{(l/2)^3 - (-l/2)^3}{3}\right]$$

$$= m\left[\frac{\left(\frac{l^3}{8}+\frac{l^3}{8}\right)}{3}\right] = \frac{m}{3}\,\frac{2}{8}\,l^3 = \frac{ml^3}{12}$$

$$I = ml.\frac{l^2}{12}$$

$$\boxed{I = \frac{Ml^2}{12}}$$

b) About an axis Passing through its one end Perpendicular to its length.



M.I of the rod about

$$PQ = Ml^2/12$$

By Parallel axis theorem

$$I = \frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2$$

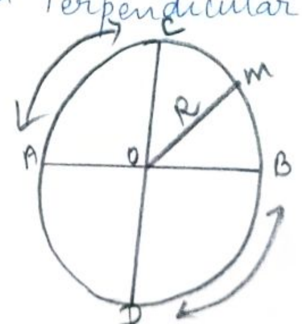
$$I = \frac{Ml^2}{12} + \frac{Ml^2}{4}$$

$$I = \frac{Ml^2 + 3l^2}{12} = \frac{4Ml^2}{12}$$

$$\boxed{I = \frac{ML^2}{3}}$$

(2) Derive an expression for M.I of the ring or loop.

a) about an axis through its centre and Perpendicular to its Plane.



m-mass, R-radius

1. AB - Passing through its centre O, Perpendicular to its plane

$I = \Sigma mR^2$

$I = MR^2$

b) About a diameter

$I_z = I_x + I_y$ —— (3)

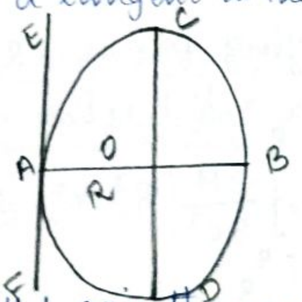
$I_z = MR^2$

$I_x = I_y = I, \quad MR^2 = I + I$

$2I = MR^2$

$\boxed{I = \frac{MR^2}{2}}$ —— (4)

c) about a tangent in the plane of the ring



By Parallel axis theorem, M.I about EF

$I = \frac{MR^2}{2} + MR^2$

$I = \frac{MR^2 + 2MR^2}{2}$

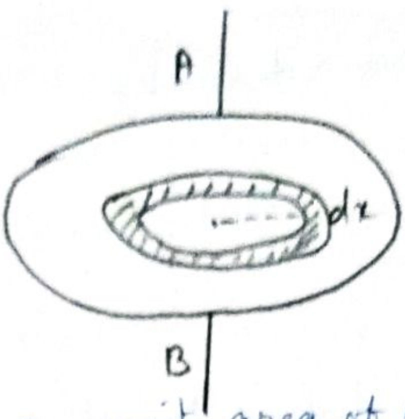
$I = \frac{3}{2}MR^2$

(3) Derive an expression for M.I of a thin circular disc.

a) About an axis through its centre and Perpendicular to its Plane

m-mass, R-radius

- AB - Passing through its centre O, ⊥r to its plane.



Mass per unit area of the disc

$= \frac{M}{\text{Area of the disc}}$

$= \frac{M}{\pi R^2}$ —— (1)

Area of the strip $= 2\pi x\,dx$

Mass of the strip $= \frac{M}{\pi R^2} 2\pi x\,dx$

$= \frac{2M}{R^2} x\,dx$ —— (2)

M.I about the axis AB

$= \frac{2M}{R^2} x^3 . dx$ —— (3)

Integrating equ (3) limits $x=0, x=R$

$I = \int_0^R \frac{2M}{R^2} x^3 dx$ —— (4)

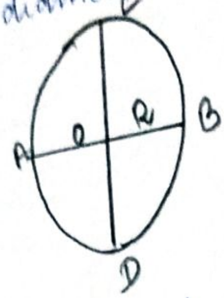
$= \frac{2M}{R^2} \int_0^R x^3 . dx$

$= \frac{2M}{R^2} \left[\frac{x^4}{4}\right]_0^R$

$= \frac{2M}{R^2} . \frac{R^4}{4}$

$\boxed{I = \frac{MR^2}{2}}$ —— (5)

b) About a diameter.



AB, CD – two ⊥r diameters AB, CD - equal M.I

$I = \frac{MR^2}{2}$ — (6)

By theorem of ⊥r axes.

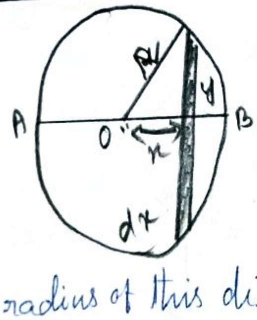
$I_z = I_x + I_y$ — (7)

$I_z = \frac{MR^2}{2}$, $I_x = I_y = I$

$\frac{MR^2}{2} = I + I \Rightarrow MR^2 = 2I$

$I = \frac{MR^2}{4}$

(4) Derive an expression for M.I of a solid sphere.



m - mass
R - radius
O - centre
dx - thickness

The radius of this disc is given by

$y^2 = (R^2 - x^2)$ — (1)

Area of the disc $= \pi y^2$ | $R^2 = x^2 + y^2$, $y = R^2 - x^2$

$= \pi (R^2 - x^2)$

Volume of the disc = Area × thickness

$= \pi (R^2 - x^2) dx$ — (2)

Mass of the elemental disc

$= \frac{3M}{4\pi R^3} \times \pi (R^2 - x^2) dx$

$= \frac{3M}{4R^3} (R^2 - x^2) dx$

M.I of disc about the axis AB

$= \frac{\text{Mass} \times (\text{Radius})^2}{2}$

$= \frac{3M}{4R^3} (R^2 - x^2) dx \times \frac{y^2}{2}$

$= \frac{3M}{4R^3} (R^2 - x^2) dx \times \frac{R^2 - x^2}{2}$

$= \frac{3M}{8R^3} (R^2 - x^2)^2 dx$ — (4)

x varying from -R to R

M.I of a solid sphere about a diameter

$I = \int_{-R}^{R} \frac{3M}{8R^3} (R^2 - x^2)^2 dx$

$= 2\int_{0}^{R} \frac{3M}{8R^3} (R^2 - x^2)^2 . dx$

$I = \frac{3M}{4R^3} \int_{0}^{R} (R^4 - 2R^2x^2 + x^4) dx$

$= \frac{3M}{4R^3} \left[R^4 x - 2R^2 \frac{x^3}{3} + \frac{x^5}{5} \right]_0^R$

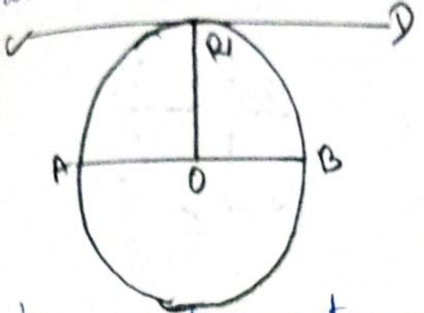
$= \frac{3M}{4R^3} \left[R^5 - \frac{2R^5}{3} + \frac{R^5}{5} \right]$

$= \frac{3M}{4R^3} \left[\frac{15R^5 + 10R^5 + 3R^5}{15} \right]$

$= \frac{3M}{4R^3} * \frac{8R^5}{15}$

$\boxed{I = \frac{2}{5} MR^2}$ — (6)

About a tangent



R - distance between tangent and diameter By Parallel axis theorem about the tangent CD.

$$I = M.I$$

$$I = \frac{2}{5}MR^2 + MR^2$$

$$I = \frac{2MR^2 + 5MR^2}{5}$$

$$\boxed{I = \frac{7}{5}MR^2}$$

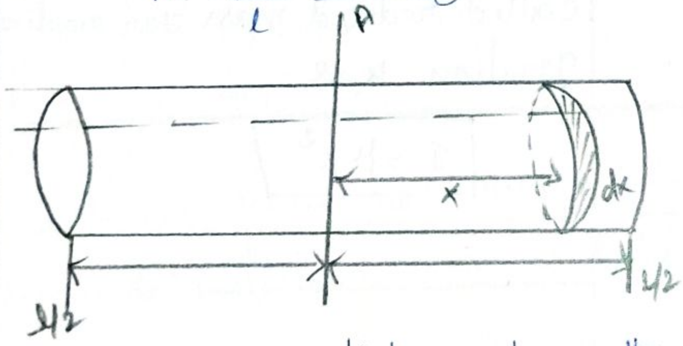
(5) <u>Derive an expression for M.I of a solid cylinder.</u>

a) About an axis passing through the centre and Perpendicular to its length

m- mass, l- length, r- radius.

Mass Per unit length of cylinder

$$m = \frac{M}{l} \quad \text{—— (1)}$$



dx - thickness, x - distance from the axis AB.

Mass of the disc = mdx

Moment of inertia of the disc about its own diameter

$$= \frac{\text{Mass} \times (\text{Radius})^2}{4} = \frac{mdx\,R^2}{4} = \frac{mR^2dx}{4}$$

using Parallel axis theorem

$$= \frac{mR^2dx}{4} + mdx\,x^2$$

Integrating above equ

$$x = -l/2,\ x = l/2$$

M.I of the cylinder about axis

$$I = \int_{-l/2}^{l/2}\left(\frac{mR^2dx}{4} + mx^2dx\right)$$

$$= \int_{-l/2}^{l/2}\frac{mR^2dx}{4} + \int_{-l/2}^{l/2} mx^2dx$$

$$= 2\int_{0}^{l/2}\frac{mR^2dx}{4} + 2\int_{0}^{l/2} mx^2dx$$

$$= \frac{mR^2}{2}(x)_0^{l/2} + 2m\left(\frac{x^3}{3}\right)_0^{l/2} \quad \text{—— (6)}$$

$$= \frac{mR^2}{2} \times l/2 + 2m \times \frac{l^3}{3\times 8}$$

$$= \frac{M}{l} \times \frac{R^2}{2} \times \frac{l}{2} + \frac{2M}{l} \times \frac{l^3}{3\times 8}$$

$$= \frac{M}{l} \times \frac{R^2}{2} \times \frac{l}{2} + \frac{2M}{l} \times \frac{l^3}{3\times 8}$$

$$I = \frac{MR^2}{4} + \frac{Ml^2}{12}$$

$$\boxed{I = M\left(\frac{R^2}{4} + \frac{l^2}{12}\right)} \quad \text{—— (7)}$$

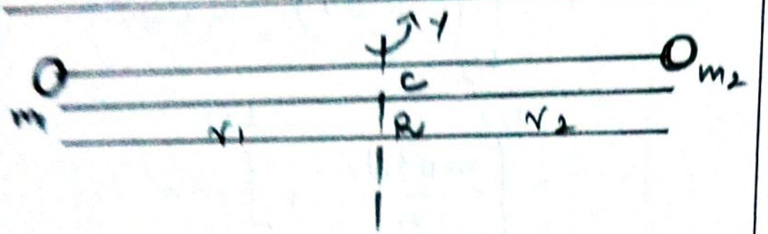
b) About the axis of the cylinder

M.I of a disc about an axis passing through its centre and Perpendicular to its Plane $= \frac{mR^2}{2}$ ———— (4)

M.I of the solid cylinder $= \Sigma \frac{mR^2}{2}$

$$\boxed{I = \frac{MR^2}{2}}$$

(8) Discuss the M.I of a diatomic molecule



• consider two masses m_1, m_2 separated by a distance R.

• C - centre of mass

r_1 & r_2 - distances of two atoms.

$$r_1 + r_2 = R \quad \text{———— (1)}$$

$$m_1 r_1 = m_2 r_2 \quad \text{———— (2)}$$

From equ (1)

$$r_1 = R - r_2 \quad \text{———— (3)}$$

From equ (2)

$$r_2 = \frac{m_1 r_1}{m_2} \quad \text{———— (4)}$$

equ (3) be

$$r_1 = R - \frac{m_1 r_1}{m_2}$$

$$R = r_1 + \frac{m_1 r_1}{m_2}$$

$$= r_1 \left[1 + \frac{m_1}{m_2}\right] \quad \text{———— (5)}$$

$$\boxed{r_1 = \frac{R}{\left(1 + \frac{m_1}{m_2}\right)}} \quad \text{———— (6)}$$

$$I = m_1 r_1^2 + m_2 r_2^2 \quad \text{———— (7)}$$

$I = m_1 r_1 \cdot r_1 + m_1 r_1 \cdot r_2$ from equ (2)

$$I = m_1 r_1 (r_1 + r_2)$$

by using equ (6)

$$I = m_1 r_1 R$$

Substituting equ (6) in (8)

$$I = m_1 R \left[\frac{R}{\left(1 + \frac{m_1}{m_2}\right)}\right]$$

$$I = \frac{m_1 R^2}{\left(1 + \frac{m_1}{m_2}\right)} = \frac{m_1 R^2}{\left(\frac{m_2 + m_1}{m_2}\right)}$$

$$I = \left(\frac{m_1 m_2}{m_1 + m_2}\right) R^2$$

$$\boxed{I = \mu R^2}$$

$\mu = \frac{m_1 m_2}{m_1 + m_2}$ - is called reduced mass for radius of gyration $K = R$

$$\boxed{I = \mu K^2}$$

...ve an expression for Angular momentum of a rigid body.

Definition

(9) Angular momentum of a Particle is defined as its momentum of linear momentum it is given by the product of linear momentum and Perpendicular distance of its line of action from the axis of rotation. It is denoted by $\vec{L}$

$$\vec{L} = \vec{r} \times \vec{p}$$

Expression for angular momentum of a rigid body.

- Rigid body rotating about a fixed fixed axis XOX'
- m_1, m_2, m_3 – masses
- r_1, r_2, r_3 – distances
- ω – angular velocity.

Angular momentum = linear momentum × distance

$= mv \times r$

$= mr\omega \times r$

$= mr^2\omega$

Angular momentum of the 1st Particle $= m_1 r_1^2 \omega$

Angular momentum of the 2nd Particle $= m_2 r_2^2 \omega$

Angular momentum of the 3rd Particle $= m_3 r_3^2 \omega$

Angular momentum of the rigid body

$= m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega$

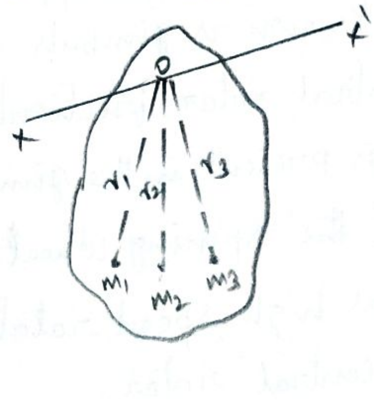
$= \omega (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2)$

$= \omega \Sigma m r^2$

$I = \Sigma m r^2$

Angular momentum of the rigid body $= \omega I$

$$\boxed{L = I\omega}$$



(10) Describe principle, construction and working of gyroscope. Mention its application in various field.

A gyroscope is a device consisting of a wheel or disc that spins rapidly about an axis that is also free to change direction

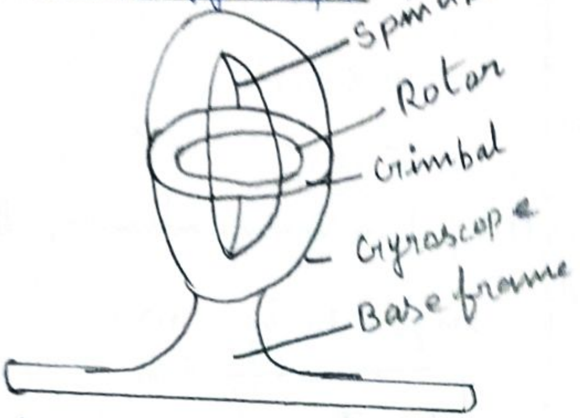
Principle: Based on conservation of angular momentum.

Properties: Two basic properties.

i) **Rigidity**: The axis of rotation of the gyrowheel tends to remain in a fixed direction in space if no force is applied to it.

ii) **Precesion**: The axis of rotation has a tendency to turn at a right angle to the direction of an applied force.

Design of gyroscope



- Rotor fixed on the supporting rings known as gimbals
- In central rotor, frictionless bearings present in the gimbals.
- Axle of the spining wheel.
- Maintains high speed rotation axis at the central rotor.
- Rotor has three degrees of rotational freedom working.
- Gimbals support the weight of the gyroscope
- cause no torques
- In axle is fixed direction, the angular momentum of the gyroscope Points along the axle.

- gyroscope used as navigation device on ships, aeroplanes and space craft.
- Need to conserve angular moment
- Gyroscope undergoes a characteristic type of motion called precession

Applications

- Used as stabilizers in ships, boats and aeroplanes
- Automatic steering systems used in airplanes and missiles.
- In gyrocompass, a directional instrument used on ships.

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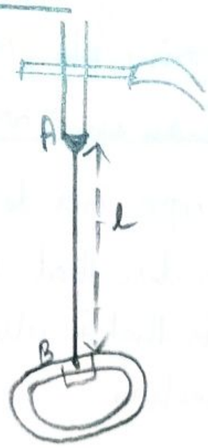
11) **Derive an expression for time Period of torsion Pendulum. Explain how it is used to find rigidity modulus of a wire.**

Definition

A circular metallic disc suspended using a thin wire that executes torsional oscillation is called torsional Pendulum.

Description



as navigation ... aeroplanes ... moment ... disc

... end fixed

lower end connected to the centre of a heavy circular disc.

Expression for the Period of oscillation of a torsion Pendulum.

- when disc is rotated by a applying twist, wire twisted through an angle θ.
- The restoring couple in the wire $= c\theta$ — (1)

c - couple Per unit twist

$$\text{Applied couple} = I\frac{d^2\theta}{dt^2}$$

At equilibrium.

applied couple = restoring couple

$$I\frac{d^2\theta}{dt^2} = -c\theta \quad \text{—— (2)}$$

Negative sign indicates the restoring couple is opposite to applied couple

$$\frac{d^2\theta}{dt^2} = -\frac{c}{I}\theta \quad \text{—— (3)}$$

The time period of oscillation

$$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$= 2\pi\sqrt{\frac{\theta}{c/I \times \theta}}$$

$$\boxed{T = 2\pi\sqrt{I/c}} \quad \text{—— (4)}$$

Uses of Torsional Pendulum

i) Rigidity modulus of the wire

ii) Moment of inertia of the disc

iii) M.I of an irregular body.

Determination of Rigidity modulus of the wire

$$\boxed{T = 2\pi\sqrt{I/c}} \quad \text{—— (1)}$$

- circular disc suspended by a thin wire.
- Top end of the wire is fixed in a vertical support.
- Disc is rotated, executes torsional oscillations.
- Time taken for 20 oscillations noted.
- Experiment repeated mean time Period is determined. The time Period of oscillation.

$$T = 2\pi\sqrt{I/c} \quad \text{—— (2)}$$

Squarring on both sides.

$$T^2 = 4\pi^2\left(\sqrt{I/c}\right)^2 \quad \text{—— (3)}$$

$$T^2 = \frac{4\pi^2 I}{c} \quad \text{—— (4)}$$

$c = \frac{\pi n r^4}{2l}$, substituting c in equ (4)

$$T^2 = \frac{4\pi^2 I}{\frac{\pi n r^4}{2l}} = \frac{2l \times 4\pi^2 I}{\pi n r^4} \quad \text{—— (5)}$$

Rearrange equ (5)

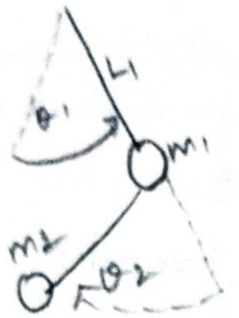
$$n = \frac{8\pi I}{r^4}\left(\frac{l}{T^2}\right)$$ — Rigidity modulus of the wire.

$$I = \frac{MR^2}{2}$$

8

(12) write notes on double Pendulum

A system in which a Pendulum is attached to the end of another Pendulum known as double Pendulum



- x - horizontal Position
- y - Vertical Position
- θ - angle of Pendulum
- L - length of rod.

- Let Position of Pendulum 1 be (x_1, y_1) Pendulum 2 (x_2, y_2)

$$x_1 = L_1 \sin\theta_1$$
$$y_1 = -L_1 \cos\theta_1$$
$$x_2 = x_1 + L_2 \sin\theta_2$$
$$y_2 = y_1 \cancel{+} - L_2 \cos\theta_2$$

The velocity is the derivative with respect to time of the Position.

$$\frac{dx_1}{dt} = \frac{d\theta_1}{dt} L_1 \cos\theta_1$$
$$\dot{x}_1 = \dot{\theta}_1 L_1 \cos\theta_1$$
$$\dot{y}_1 = \dot{\theta}_1 L_1 \sin\theta_1$$
$$\dot{x}_2 = \dot{x}_1 + \dot{\theta}_2 L_2 \cos\theta_2$$
$$\dot{y}_2 = \dot{y}_1 + \dot{\theta}_2 L_2 \sin\theta_2$$

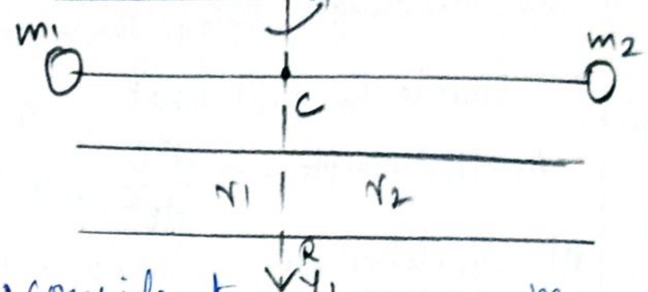
The acceleration is the second derivative

$$\ddot{x}_1 = -\dot{\theta}_1^2 L_1 \sin\theta_1 + \ddot{\theta}_1 L_1 \cos\theta_1$$
$$\ddot{y}_1 = \dot{\theta}_1^2 L_1 \cos\theta_1 + \ddot{\theta}_1 L_1 \sin\theta_1$$
$$\ddot{x}_2 = \ddot{x}_1 - \dot{\theta}_2^2 L_2 \sin\theta_2 + \ddot{\theta}^2 L_2 \cos\theta_2$$
$$\ddot{y}_2 = \ddot{y}_1 \dot{\theta}_2^2 L_2 \cos\theta_2 + \ddot{\theta}^2 L_2 \sin\theta_2$$

Uses of double Pendulum:

- Used in education, research applications.
- Used to study chaos both experimentally and numerically

(13) Discuss the rotational energy states of a rigid diatomic molecule.



- consider two Masses m_1, m_2
- r_1, r_2 Distance
- yy' axis of rotation
- Arrangement called rigid rotor
- R - bond length between two atoms. $(R = r_1 + r_2)$

K.E is given as

$$E = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 \quad (1)$$
$$E = \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2)\omega^2$$
$$E = \frac{1}{2} I \omega^2$$

M.I $\quad I = m_1 r_1^2 + m_2 r_2^2$

$$E = \frac{1}{2} I \omega^2 \quad (2)$$

Equ (2) rewritten as

$$E = \frac{1}{2I} \cdot I^2 \omega^2 \quad (3)$$
$$I\omega = L$$

(3) becomes.

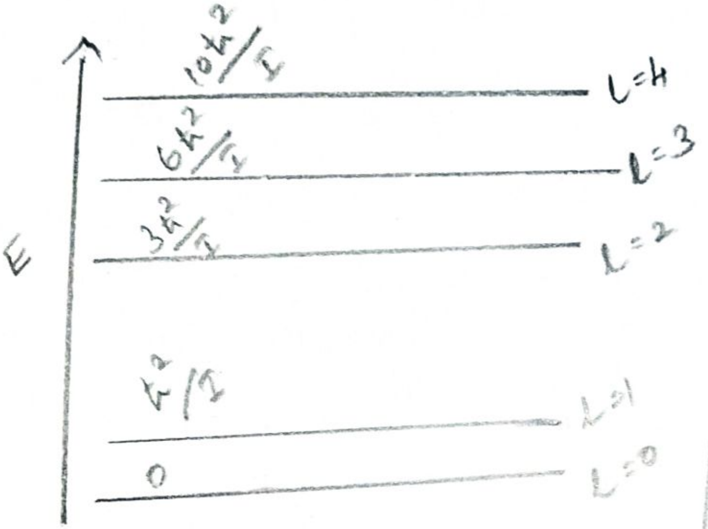
$$E = \frac{L^2}{2I} \quad \text{——— (4)}$$

$$L^2 = l(l+1)\hbar^2 \qquad l = 0, 1, 2, \ldots \quad (5)$$

l - rotational quantum number
l varies interms of integer values

$$E_l = \frac{l(l+1)\hbar^2}{2I} \quad \text{——— (6)}$$

$\hbar = \frac{h}{2\pi}$ $\quad$ h - Planck's constant.



Levels are not equally spaced.

UNIT - II

Electromagnetic waves

The Maxwell's equs - wave equ: Plane electromagnetic waves in a vaccum, conditions on the wave field - Properties of electromagnetic waves: Speed, amplitude, Phase, orientation and waves in matter - Polarization - Producing electromagnetic waves - Energy and momentum in EM waves: Intensity waves from localized sources momentum and radiation pressure - cell phone reception, Reflecting and transmission of electromagnetic waves from a non-conducting medium vaccum interface for normal incidence.

① **Derive Maxwell's equ's in differential and Integral form**

Maxwell's equ - I (From Gauss law in electrostatics.

Gauss law in electrostatics state that the total electric flux through any closed surface is equal to the charge enclosed by it

According to Gauss law

$$\oint\!\!\oint_S \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon} \quad \text{—— ①}$$

$$\oint\!\!\oint_S \varepsilon\,\vec{E}\cdot\vec{ds} = q \qquad (\vec{D} = \varepsilon\vec{E})$$

$$\oint\!\!\oint \vec{D}\cdot\vec{ds} = q \quad \text{—— ②}$$

Total charge inside the closed surface is

$$q = \iiint_V \rho\, dV \quad \text{—— ③}$$

substituting equ ③ in ②

$$\oint\!\!\oint \vec{D}\cdot\vec{ds} = \iiint_V \rho\, dV \quad \text{—— ④}$$

equ ④ is the Maxwell's equ in integral form from Gauss law in electrostatics.

Applying Gauss divergence theorem to LHS of equ ④

$$\oint\!\!\oint_S \vec{D}\cdot\vec{ds} = \iiint_V \vec{\nabla}\cdot\vec{D}\, dV \quad \text{—— ⑤}$$

on substituting equ ⑤ in equ ④

$$\iiint_V \vec{\nabla}\cdot\vec{D}\, dV = \iiint_V \rho\, dV \quad \text{—— ⑥}$$

(or) $\vec{\nabla}\cdot\vec{D} = \rho$

$$\vec{\nabla}\varepsilon_0\vec{E} = \rho \qquad (D = \varepsilon_0\vec{E})$$

$$\varepsilon_0\,\vec{\nabla}\cdot\vec{E} = \rho$$

$$\overrightarrow{\nabla\cdot E} = \rho/\varepsilon_0$$

This is Maxwells equ from Gauss law in electrostatics in differential form.

Maxwells equ II [from Gauss's law in magnetostatics

Integral form

Total magnetic flux through any closed surface in a magnetic field is zero.

$$\oiint \vec{B}.\vec{ds} = 0 \quad \text{---} (1)$$

This is maxwells equ in integral form from Gauss law in magneto-statics.

L.H.S of equ (1)

$$\oiint \vec{B}.\vec{ds} = \iiint_V \vec{\nabla}.\vec{B}\, dV \quad \text{---} (2)$$

Substituting equ (2) in (1)

$$\iiint_V \vec{\nabla}.B dV = 0 \quad \text{---} (3)$$

(or) $\vec{\nabla}.\vec{B} = 0$ ——— (4)

This is Maxwells equ in differential form from Gauss's law in magnetostatics.

Maxwells equ III (From Faraday's law)

Magnetic flux through a small area $ds = \vec{B}.\vec{ds}$ ——— (1)

Total magnetic flux linked with the circuit $\phi_B = \oiint_S \vec{B}.\vec{ds}$ ——— (2)

Faraday's law states that the induced emf e is the rate of change of magnetic flux. ϕ_B

$$e = \frac{-d\phi_B}{dt} = \frac{-d}{dt}\left[\oiint \vec{B}.\vec{ds}\right] \quad \text{---} (3)$$

$$= \iint \frac{\partial B}{\partial t} = ds$$

$\vec{E}$ - Electric field strength

$\vec{E} = \frac{dV}{dl}$, $dV = \vec{E}.\vec{dl}$

$V = \int dV = \int \vec{E}.\vec{dl}$

$V = e = \int \vec{E}.\vec{dl}$

$e = \oint \vec{E}.\vec{dl}$ ——— (4)

Equating equ (3) in equ (4)

$$\oint_C \vec{E}.\vec{dl} = -\iint_S \frac{\partial \vec{B}}{\partial t} = \vec{ds} \quad \text{---} (5)$$

This is Maxwell's equ in integral form from Faraday's law of electromagnetic induction

Applying stokes theorem to LH.S equ (5)

$$\oint_C \vec{E}.\vec{dl} = \iint_S (\vec{\nabla}.\vec{E}).\vec{ds} \quad \text{---} (6)$$

Substituting equ (6) in (5)

$$\iint_S (\vec{\nabla} \times \vec{E})\, d\vec{S} = -\iint \frac{\partial \vec{B}}{\partial t} . \vec{ds} \quad \text{---} (7)$$

$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \quad \text{---} (8)$$

From equ (8) represents Maxwells equ from Faraday's law of Electromagnetic induction in different form.

Statement:

The electromotive force around a closed path is equal to the rate of magnetic displacement through the closed path.

Maxwells equ IV

From Amperes circutal law.

Amperes law states that the line integral of magnetic field intensity H on any closed Path is equal to the current (I) enclosed by that path.

$$\oint \vec{H}\cdot d\vec{l} = I \quad \text{--- (1)}$$

$J = I/A$, $I = JA$

$$I = J\iint_S ds$$

$$I = \iint_S \vec{J}\cdot ds \quad \text{--- (2)}$$

Substituting equ (2) in (1)

$$\oint \vec{H}\cdot d\vec{s} = \iint_S \vec{J}\cdot d\vec{s} \quad \text{--- (3)}$$

Amperes law is modified by introducing displacement current density.

$$\oint \vec{H}\cdot d\vec{l} = \iint_S (\vec{J_c} + \vec{J_D})\, d\vec{s} \quad \text{--- (4)}$$

Substituting $\vec{J_c} = \sigma E$,

$$\vec{J_D} = \frac{\partial \vec{D}}{\partial t}$$

$$\oint \vec{H}\cdot d\vec{s} = \iint_S \left(\sigma\vec{E} + \frac{\partial \vec{D}}{\partial E}\right) ds$$

$$\oint \vec{H}\cdot d\vec{l} = \iint_S \left(\sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t}\right) ds \quad \text{--- (5)}$$

$$\oint \vec{H}\cdot d\vec{l} = \iint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) ds \quad \text{--- (6)}$$

$(J = \sigma E,\ D = \varepsilon\vec{E})$

This is Maxwells equ in integral form from Ampere's circuital law.

Applying stokes theorem to L.H.S of equ (6)

$$\oint \vec{H}\cdot d\vec{l} = \iint_S (\vec{\nabla}\cdot\vec{H})\, ds \quad \text{--- (7)}$$

Substituting equ (7) in (6)

$$\iint_S (\vec{\nabla}\times\vec{H})\, ds = \iint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) ds \quad \text{--- (8)}$$

$$(\vec{\nabla}\times\vec{H}) = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- (9)}$$

$$\vec{\nabla}\times\vec{H} = \sigma E + \varepsilon\frac{\partial \vec{E}}{\partial t} \quad \text{--- (10)}$$

Equs (9) + (10) are Maxwells equs in differential form from Amperes circutal law.

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(2) Deduce Maxwells equs for free space

Four Maxwells equs for free space

$$\vec{\nabla}\cdot\vec{D} = \rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- (4)}$$

Maxwells equs reduce to

$$\vec{\nabla}\cdot\vec{D} = 0 \quad \text{--- (5)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{--- (6)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (7)}$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial D}{\partial t} \quad \text{--- (8)}$$

Maxwells equ in conducting media

$\vec{J} = \sigma\vec{E}$ $\quad \vec{D} = \varepsilon\vec{E}$, ε Permittivity

$\vec{B} = \mu\vec{H}$

General Maxwell equs reduced to

$$\vec{\nabla}\cdot\vec{D} = \rho \quad \text{--- (1)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{--- (2)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (3)}$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- (4)}$$

and optically flat and Parallel

. Elliptical

(3) Explain the characteristics of Maxwells equ

Maxwell first equ $\vec{\nabla}.\vec{D} = \rho$

i) Maxwell first equ

. Explain Gauss's law in electrostatics

. Time independent or steady state equ

. The flux of the lines of electric forces depends upon charge density.

. charge acts as a source.

ii) Maxwells second equ $\vec{\nabla}.\vec{B} = 0$

. Explain Gauss law in magnetostatics

. Time independent equ.

. No Source.

iii) Maxwells third equ $\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t}$

. It explains Faraday's Law + Lens law

. Time independent equ.

. $\vec{E}$ is generated by the time variation of $\vec{B}$

iV) Maxwell's Fourth equ

$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$$

. Relation with magnetic field vector $\vec{B}$, with displacement Vector $\vec{D}$ and the current density.

. Time dependent equ

. Explains Ampere's circuital law.

(4) Discuss the Plane wave

Definition

If a wave is confined to a Particular axis with equal magnitudes of electric and magnetic field vectors then the wave is called Plane wave.

Plane Electromagnetic wave equ in Vaccum

Maxwells equ in general form

$\vec{\nabla}.\vec{D} = \rho$ — (1) $\quad \vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t}$ — (3)

$\vec{\nabla}.\vec{B} = 0 \quad \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$ — (4)

$\vec{D} = \varepsilon_0\vec{E}$, $\vec{B} = \mu_0\vec{H}$

conductivity $\sigma = 0$

$\vec{J} = 0$

No charge Present in the vaccum

$\rho = 0$ equ (1) reduces to

$\vec{\nabla}.\vec{D} = 0$ (or)

$\vec{\nabla}.\varepsilon_0\vec{E} = 0$

$\varepsilon_0\vec{\nabla}.\vec{E} = 0$

$$\boxed{\vec{\nabla}.\vec{E} = 0}$$

wave equ for electric field Vector ($\vec{E}$)

Taking curl on both sides of equ (3)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(\frac{-\partial\vec{B}}{\partial t}\right)$$

$$= \frac{-\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$= \frac{\partial}{\partial t}(\vec{\nabla}\times\mu_0\vec{H})$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu_0 \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})$$

Now from vector calculus identity

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E}$$

from equ (5)

$$\vec{\nabla}\cdot\vec{E} = 0$$

Substituting this in equ (7)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\nabla^2\vec{E} \quad \text{——— (8)}$$

Substituting equ (8) in (6)

$$-\nabla^2 E = -\mu_0 \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})$$

Substituting for $\vec{\nabla}\times\vec{H}$ from equ (4)

$$-\nabla^2 E = -\mu_0 \frac{\partial}{\partial t}\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

$$-\vec{\nabla}^2\vec{E} = -\mu_0 \frac{\partial}{\partial t}\left[\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right]$$

$$\boxed{\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \quad \text{——— (10)}$$

Equ (10) is the general electro magnetic wave equ.

<u>wave equ for magnetic field vector ($\vec{B}$)</u>

Taking curl on both sides of the equ (4)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\times\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \varepsilon_0 \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$$

From vector calculus identity

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \nabla^2\vec{H}$$

From equ (2)

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\mu_0(\vec{\nabla}\cdot\vec{H}) = 0 \text{ or } \vec{\nabla}\cdot\vec{H} = 0$$

Substituting equ (13) in equ (12)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = -\nabla^2\vec{H} \quad \text{——— (14)}$$

using equ (14) and equ (11)

$$-\nabla^2\vec{H} = \varepsilon_0 \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$$

Substituting the equ (3) in equ (15)

$$-\nabla^2\vec{H} = \varepsilon_0 \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) \quad \text{——— (15)}$$

$$-\nabla^2\vec{H} = \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{-\partial \vec{B}}{\partial t}\right)$$

$$-\nabla^2 H = -\varepsilon_0 \frac{\partial^2}{\partial t^2}(\mu_0 \vec{H})$$

$$\boxed{\nabla^2\vec{H} = \mu_0\varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}} \quad \text{——— (16)}$$

This general electromagnetic wave equ interms of $\vec{H}$ for free space.

The electromagnetic wave equ for $\vec{E}$ and $\vec{H}$ is written as

$$\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{——— (17)}$$

$$\nabla^2 H - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad \text{——— (18)}$$

In one dimension say along x-axis the wave equs are given by the x-component of the above expression

$$\frac{\partial^2 E_x}{\partial x^2} - \mu_0\varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} = 0 \quad \text{——— (19)}$$

$$\frac{\partial^2 H_x}{\partial x^2} - \mu_0\varepsilon_0 \frac{\partial^2 H_x}{\partial t^2} = 0 \quad \text{——— (20)}$$

Speed of EM wave in Vaccum

comparing equ (19) and (20)

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t} = 0 \quad \text{——— (21)}$$

Laser

y - instantaneous displacement
c - velocity of wave.
The velocity of the electromagnetic wave is given by.

$$\frac{1}{c^2} = \mu_0 \varepsilon_0 \quad c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Magnitude of velocity is called

Speed $c = 1/\sqrt{\mu_0 \varepsilon_0}$ —— (22)

For vacum or free space

$$\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$$

$$\varepsilon_0 = 8.842 \times 10^{-12}\ Fm^{-1}$$

Substituting these values in equ (22)

$$\boxed{c = 2.998 \times 10^8\ ms^{-1}}$$

wave equs for Plane Polarised EM wave in free space are given by

The EM wave equs.

$$\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{—— (1)}$$

$$\nabla^2 H - \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} = 0 \quad \text{—— (2)}$$

conditions on the wavefield

If the plane Polarized waves is propagating along x-axis having electric vector along the y-axis

$E_y \neq 0, \ E_z = E_x = 0$

For magnetic field vector.

$H_z \neq 0 \quad H_y = H_x = 0$

The wave equs for plane electro magnetic wave reduced to.

$$\nabla^2 E_y - \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} = 0 \quad \text{—— (3)}$$

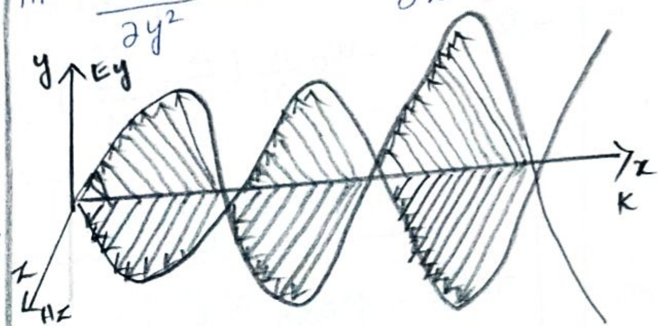
$$\nabla^2 H_z - \mu_0 \varepsilon_0 \frac{\partial^2 H_z}{\partial t^2} = 0 \quad \text{—— (4)}$$

$$\nabla^2 E_y = \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} \quad \text{—— (5)}$$

$$\nabla^2 H_z = \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} \quad \text{—— (6)}$$

$$\frac{\partial^2 E_y}{\partial y^2} = 0 \text{ and } \frac{\partial^2 E_y}{\partial z^2} = 0$$

$|||^{ly}$ $\frac{\partial^2 H_z}{\partial y^2} = 0$ and $\frac{\partial^2 H_z}{\partial z^2} = 0$



$$\nabla^2 E_y = \frac{\partial^2 E_y}{\partial x^2} \quad \nabla^2 H_z = \frac{\partial^2 H_z}{\partial x^2}$$

Substituting equ (7) in equ (3) and equ (8) in equ (4)

$$\frac{\partial^2 E_y}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} = 0 \quad \text{—— (9)}$$

$$\frac{\partial^2 H_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 H_z}{\partial t^2} = 0 \quad \text{—— (10)}$$

Solutions of the plane wave equs.

The plane wave equs for electric field and magnetic field are gn by.

(I) $$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 H_z}{\partial t^2} = 0$$

c - Speed of EM wave

The solutions of the above wave equs of progressive wave are gn by

$$E_y = E_0 \cos(\omega t - kx) \quad \text{——(11)}$$

$$H_z = H_0 \cos(\omega t - kx) \quad \text{——(12)}$$

The general solution of the wave equ is written as

$$\vec{E_y} = E_0 e^{i(\omega t - kx)} = E_0 e^{ik(ct-x)}$$

$$H_z = H_0 e^{i(\omega t - kx)} = H_0 e^{ik(ct-x)}$$

where

$$k = \frac{2\pi}{\lambda},\ \omega = 2\pi\nu = \frac{2\pi c}{\lambda} = kc$$

c = wave velocity.

(5) <u>Explain Phase and orientation of EM wave in matter.</u>

Electric and Magnetic fields are same $(\omega t - kx)$. Both fields are in Phase with each other.

Relation between electric and magnetic field vectors.

For electromagnetic waves in free space.

$$\vec{E_y} = E_0 e^{ik(ct-x)} \quad \text{——(1)}$$

$$H_z = H_0 e^{ik(ct-x)} \quad \text{——(2)}$$

The relation between their time and space variations is given from Maxwells equ

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\text{(or)}\ \frac{\partial \vec{E_y}}{\partial x} = -\mu_0 \frac{\partial \vec{H_z}}{\partial t} \quad \text{——(3)}$$

Substituting E_y and H_z from the equs (1) & (2) in equ (3)

$$\frac{\partial}{\partial x}\left(E_0 e^{ik(ct-x)}\right) = -\mu_0 \frac{\partial}{\partial t}\left(H_0 e^{ik(ct-x)}\right) \quad \text{——(4)}$$

$$-ikE_0 e^{ik(ct-x)} = -\mu_0(ikc).H_0 e^{ik(ct-x)}$$

$$E_0 = \mu_0 c H_0 \quad \text{——(5)}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \text{——(6)}$$

Substituting equ (6) in equ (5)

$$E_0 = \mu_0 \times \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \times H_0 = \sqrt{\mu_0/\varepsilon_0}\,.H_0$$

$$\sqrt{\mu_0/\varepsilon_0} = \frac{E_0}{H_0} = \frac{E_0 e^{i(\omega t - kx)}}{H_0 e^{i(\omega t - kx)}}$$

$$\frac{\vec{E}}{\vec{H}} = \sqrt{\mu_0/\varepsilon_0} \quad \text{——(7)}$$

This is the relation between the electric field vector and magnetic field vector.

$$\frac{\vec{E}}{\vec{H}} = \frac{E_0}{H_0} = \sqrt{\mu_0/\varepsilon_0}$$

The ratio E/H is having the unit of impedence ie) ohm. The quantity $\sqrt{\mu_0/\varepsilon_0}$ has the dimensions of impedence

$$\sqrt{\mu_0/\varepsilon_0} = \sqrt{\frac{H/m}{F/m}} = \sqrt{\frac{henry/m}{Farad/m}}$$

$$\sqrt{\frac{henry}{Farad}} = \frac{ohm \times sec}{Joul/volt} = \sqrt{\frac{ohm \times volt}{Joule/sec}}$$

$$= \sqrt{\frac{amp \times volt}{ohm}} = \sqrt{ohm \times ohm} = ohm.$$

It is known as intrinsic or characteristic impedence of free space, denoted by Z_0. $\vec{E}$ is parallel to Y-axis. The vector $(\vec{E} \times \vec{H})$ is known as Poynting vector.

⑥ <u>what is mean by Poynting vector? what is it significance.</u>

The cross product of electric field vector $\vec{E}$ and the magnetic field vector $\vec{H}$ is called Poynting vector. It is denoted by

$$\vec{S} = \vec{E} \times \vec{H}$$

- A plane Polarized electromagnetic wave is propagated along the x-axis
- Electric vector is directed along the y-axis
- Magnetic vector is directed along the z-axis.

$$\vec{S} = \vec{E} \times \vec{H} = \hat{j} E_y + \hat{k} H_z$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & E_y & 0 \\ 0 & 0 & H_z \end{vmatrix} = \hat{i}(E_y H_z)$$

- Poynting vector gives the time ratio of flow of electromagnetic wave energy per unit area of the medium.
- The average Poynting vector for one complete cycle of electromagnetic wave is given by

$$S_{avg} = \frac{1}{2}(\vec{E} \times \vec{H})$$

$$= \frac{1}{2} E_0 \times H_0$$

$$= \frac{E_0}{\sqrt{2}} \times \frac{H_0}{\sqrt{2}} = E_{rms} H_{rms}$$

<u>Significance</u>

- If there is a varying electric field in vaccum, there is also a varying magnetic field.
- Electric and magnetic fields obey wave eqn.

the speed of propagation given by $c = 1/\sqrt{\mu_0 \varepsilon_0}$ is the Same as the measured speed of Light.

. Light waves, be identified as electromagnetic waves.

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(7) Discuss propagation of electro magnetic wave through a Dielectric medium.

Maxwell's equ are

$$\vec{\nabla}\cdot\vec{D} = \rho \qquad \vec{\nabla}\times\vec{E} = \frac{-\partial B}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

In an isotropic dielectric

$$\vec{D} = \varepsilon\vec{E} \quad \vec{B} = \mu\vec{H}, \quad \vec{J} = \sigma\vec{E} = 0 \text{ and } \rho = 0$$

There fore Maxwell's equ

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{—— (1)}$$

$$\vec{\nabla}\cdot\vec{H} = 0 \quad \text{—— (2)}$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial B}{\partial t} \quad \text{—— (3)}$$

$$\vec{\nabla}\times\vec{H} = \varepsilon\frac{\partial E}{\partial t} \quad \text{—— (4)}$$

Equation of propagation of Magnetic vector H.

Taking curl of equ (4)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \varepsilon\,\vec{\nabla}\times\left(\frac{\partial \vec{E}}{\partial t}\right)$$

(or)

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \nabla^2\vec{H} = \varepsilon\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) \quad \text{—— (5)}$$

Putting values from the equs (2) and (3)

$$\nabla^2 H = \mu\varepsilon\frac{\partial^2 \vec{H}}{\partial t^2} \quad \text{—— (6)}$$

Equation of propagation of electric vector, E

Taking of equ (3)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\mu\frac{\partial \vec{H}}{\partial t}\right)$$

$$\nabla(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})$$

Putting values from equs (1) and (4)

$$\nabla^2 E = \mu\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{—— (7)}$$

The equs (6) and (7) compared with general wave equ.

$$\nabla^2 u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = c, \text{ speed of electromagnetic}$$

Refractive index is

$$n = \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} = \sqrt{\mu_r\varepsilon_r}$$

In a non-magnetic medium $\mu_r = 1$

$$n = \sqrt{\varepsilon_r}$$

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(8) Discuss electromagnetic wave in conducting finite μ, ε and σ)

General Maxwells equ

$$\vec{\nabla} \cdot \vec{D} = \rho \quad (1)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (2)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (4)$$

In conducting medium $\sigma \neq 0$

equ (1) reduces to $\vec{\nabla} \cdot \vec{D} = 0$

$\vec{\nabla} \cdot (\varepsilon \vec{E}) = 0$, $\vec{\nabla} \cdot \vec{E} = 0$

$$\vec{D} = \varepsilon \vec{E} \quad (5)$$

Taking the curl on both ~~sng~~ sides of equ (3)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(\frac{-\partial \vec{B}}{\partial t}\right) \quad (6)$$

From vector calculus identity

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E})$$
$$= \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} \quad (7)$$

But from equ (5) $\vec{\nabla} \cdot \vec{E} = 0$

Therefore equ (7) becomes.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\nabla^2 \vec{E} \quad (8)$$

Also

$$\vec{\nabla} \times \left(\frac{-\partial \vec{B}}{\partial t}\right) = -\mu \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H}) \quad (9)$$

Substituting the equ (8) and equ (9) in (6)

$$-\nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H})$$

(or)

$$\nabla^2 E = \mu \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H}) \quad (10)$$

Substituting the value of $\vec{\nabla} \times \vec{H}$ from equ (4) in equ (10)

$$\nabla^2 E = \mu \frac{\partial}{\partial t}\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \quad (11)$$

Since $\vec{J} = \sigma \vec{E}$ and $\vec{D} = \varepsilon \vec{E}$

equ (11) becomes

$$\nabla^2 \vec{E} = \mu \frac{\partial}{\partial t}\left[\sigma \vec{E} + \frac{\partial}{\partial t}(\varepsilon \vec{E})\right] \text{ (or)}$$

$$\nabla^2 E = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}}{\partial t} = 0 \quad (12)$$

This is the general wave equ for the electric vector in an electromagnetic wave propagating in conducting medium

By taking curl of the equ (4)

$$\nabla^2 H - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} - \mu\sigma \frac{\partial \vec{H}}{\partial t} = 0 \quad (13)$$

wave equ for Plane Polarized EM waves.

consider electro magnetic wave is travelling in the x-direction and the electric vector is directed along the Y-axis and the magnetic vector is directed along the z-axis

$y \neq 0$, $E_x = E_z$ and $H_x \neq 0, H_y = H_x = 0$

wave equs from (12) and (13)

$$\frac{\partial^2 E_y}{\partial x^2} - \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2} - \mu\sigma \frac{\partial E_y}{\partial t^2} = 0 \quad \text{——— (14)}$$

$$\frac{\partial^2 H_z}{\partial x^2} - \mu\varepsilon \frac{\partial^2 H_z}{\partial t^2} - \mu\sigma \frac{\partial H_z}{\partial t} = 0 \quad \text{——— (15)}$$

above equs $\mu\varepsilon = \frac{1}{\nu^2}$

ν - velocity of electromagnetic wave

The product $\mu\sigma$ is called magnetic diffusivity

__Solution of the Plane EM wave equ in conduction medium ($\sigma \neq 0$)__

equ (14) in the function of t and in the form

$$\vec{E_y} = E_0 e^{(i\omega t \pm \nu x)} \quad \text{——— (16)}$$

solution of equ (15)

$$\vec{H_z} = H_0 e^{(i\omega t \pm \nu x)}$$

$$-\mu\varepsilon \frac{\partial^2}{\partial t^2} (E_0 e^{i\omega t \pm \nu x}) - \mu\sigma \frac{\partial}{\partial t} (E_0 e^{(i\omega t \pm \nu x)}) = 0$$

$$\nu^2 E_0 e^{(i\omega t \pm \nu x)} - \mu\varepsilon (i\omega)^2 E_0 e^{(i\omega t \pm \nu x)} - \mu\sigma i\omega E_0 e^{(i\omega t \pm \nu x)} = 0$$

$$\nu^2 - \mu\varepsilon i^2 \omega^2 - \mu\sigma i\omega = 0$$

$$\nu^2 + \mu\varepsilon\omega^2 - i\mu\sigma\omega = 0$$

$$\nu^2 = i\mu\sigma\omega - \mu\varepsilon\omega^2 \quad \text{——— (18)}$$

$\mu\varepsilon\omega^2$ can be neglected as compared to $\mu\sigma\omega$ from equ (18)

$$\nu^2 = i\mu\sigma\omega$$

$$\nu^2 = \frac{2i\mu\sigma\omega}{2} \quad \text{(or)}$$

Taking square root on both sides

$$\nu = \pm(1+i)\sqrt{\frac{\mu\sigma\omega}{2}}$$

$$\nu = (1+i)K \text{ (or)}$$

$$\nu = -(1+i)K$$

$K = \sqrt{\frac{\mu\sigma\omega}{2}}$ is a constant - taking negative values of ν which gives the wave propagation in the positive x direction,

Substituting in equ (16)

$$\vec{E_y} = E_0 e^{(i\omega t - (1+i)Kx)}$$

$$\vec{E_y} = E_0 e^{(i\omega t - Kx - iKx)}$$

$$\vec{E_y} = E_0 e^{-Kx} e^{i(\omega t - Kx)}$$

This is a progressive wave having amplitude equal to $E_0 e^{-Kx}$.

__(9) Determine skin depth in conducting Material (or) Penetration depth.__

In conducting medium amplitude of the electromagnetic wave decreases exponentially with distance of Penetration of the wave.

The amplitude of a depth x is denoted by E_{0x}

$$E_{0x} = E_0 e^{-Kx} \quad \text{——— (1)}$$

$$K = \sqrt{\frac{\mu \sigma \omega}{2}}$$

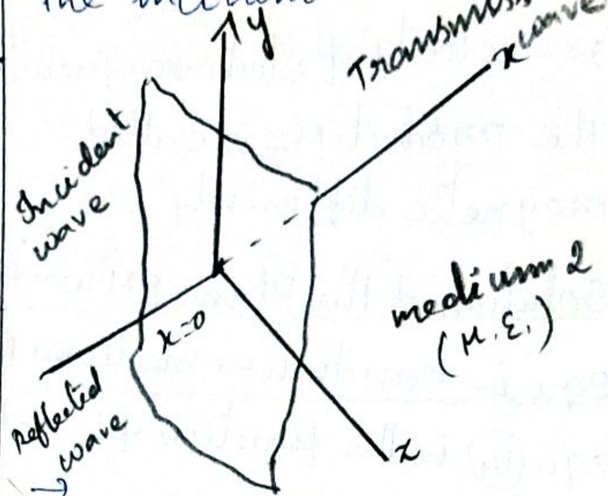
It is defined as the distance inside the conductor from the surface of the conductor at which the amplitude of the field vector is reduced to 1/e times its value at the surface.

(10) Discuss the properties of Electromagnetic waves.

- Electromagnetic waves are transverse in nature.
- Produced by accelerated charges.
- EM waves travel with speed of light and doesn't need any medium to propagate.
- EM waves are not deflected by electric or magnetic field.
- Exhibit interference or diffraction and can be Polarized.
- EM waves being chargeless.
- The energy in an EM wave is equally divided between electric and magnetic field vector.

(11) Discuss pro... from vaccum to a non-co... medium.

- A monochromatic uniform pla... wave travels through one medium and enters another medium.
- Incoming EM wave is called the incident wave.



$$\vec{E}_i(x,t) = E_0 \cos(\omega t - K_1 x) \quad (1)$$

$$\vec{B}_i(x,t) = \frac{E_0}{v_1}\cos(\omega t - K_1 x) \quad (2)$$

Reflected waves are represented.

$$E_R(x,t) = E_1 \cos(\omega t + K_1 x) \quad (3)$$

$$B_R(x,t) = \frac{E_1}{v_1}(\cos \omega t + K_1 x) \quad (4)$$

$$\vec{E}_T(x,t) = E_2 \cos(\omega t - K_2 x) \quad (5)$$

$$B_T(x,t) = \frac{E_2}{v_2}\cos(\omega t - K_2 x) \quad (6)$$

equ (3) and (4) sign is reversed in the wave number K, along negative x direction K_1, K_2 wave num being

$$K_1 = \omega / v_1 \quad (7)$$

$$K_2 = \omega / v_2 \quad (8)$$

v_1, v_2 velocity.

total instantaneous electric field $\vec{E_y}$

$$E_y(x,t) = E_0\cos(\omega t - k_1x) + E_1\cos(\omega t + k_1x) \quad \text{—— (9)}$$

$$E_y(x,t) = \vec{E_i}(x,t) + E_R(x,t) \quad \text{—— (10)}$$

$$E_y(x,t) = E_2\cos(\omega t - k_2x) \quad \text{—— (11)}$$

At interface $x=0$

Since the waves are transverse $\vec{E},\vec{B}$ fields tangential to the interface

$$E_0\cos(\omega t - k_1x) + E_1\cos(\omega t - k_1x) = E_2\cos(\omega t - k_2x) \quad \text{—— (12)}$$

$$E_0\cos(\omega t) + E_1\cos(\omega t) = E_2\cos(\omega t)$$

$$E_0 + E_1 = E_2 \quad \text{—— (13)}$$

At boundary $x=0$,

$$\frac{dE_i}{dx} + \frac{dE_R}{dx} = \frac{dE_T}{dx} \quad \text{—— (14)}$$

$$-E_0k_1\sin(\omega t) - E_1k_1\sin(\omega t) = E_2k_2\sin(\omega t) \quad \text{—— (15)}$$

$$E_0k_1 - E_1k_1 = E_2k_2$$

$$k_1(E_0 - E_1) = E_2k_2$$

$$E_0 - E_1 = E_2(k_2/k_1) \quad \text{—— (16)}$$

$k_1 = \omega/v_1$, $k_2 = \omega/v_2$

then equ (16)

$$E_0 - E_1 = E_2(v_1/v_2) \quad \text{—— (17)}$$

Adding equ (13) & (17)

$$2E_0 = E_2 + E_2(v_1/v_2)$$

$$= E_2(1 + v_1/v_2)$$

$$E_0 = \frac{E_2}{2}(1 + v_1/v_2) \quad \text{—— (18)}$$

when medium -1,

Vaccum $v_1 = c$, $v_2 = v$

$$E_0 = \left(\frac{E_2}{2}\right)\left(1 + \frac{c}{v}\right)$$

Subtracting equ (17) from equ (18)

$$E_1 = \frac{E_2}{2}(1 - v_1/v_2) \quad \text{—— (19)}$$

$$E_1 = \left(\frac{E_2}{2}\right)\left(1 - \frac{c}{v}\right)$$

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(12) write short notes on

i) Momentum and Radiation Pressure

ii) cell phone Reception

i) Momentum and Radiation Pressure

- Electromagnetic waves carry energy and momentum.
- Maxwell proved wave energy v.
- Momentum are related by $p = V/c$ —— (1)

As the electromagnetic waves carry momentum, they exert pressure when they are reflected or absorbed at the surface of a body. This is known as radiation Pressure.

From Newton's second law, change in momentum related to force.

$$F = \frac{\Delta P}{\Delta t} \quad \text{—— (2)}$$

Intensity $I = \frac{\text{Power}}{\text{area}} = \frac{\text{energy/time}}{\text{area}}$

$\Delta U = I \cdot A \cdot \Delta t$ —— (3)

equ (1) momentum.

$\Delta P = \frac{\Delta U}{c} = \frac{I \cdot A \cdot \Delta t}{c}$ —— (4)

$F = \frac{\Delta P}{\Delta t} = I \cdot A/c$ —— (5)

This is the relation for the total absorption of EM radiation

ΔP - change in momentum.

$F = 2IA/c$ —— (6)

If the radiation is Party absorbed or completely reflected by the object, the magnitude of the force on area A varies between the values IA/c and $2IA/c$

Radiation Pressure.

The force Per unit area on an object due to EM radiation is the radiation pressure P_r

From equ (5) and (6)

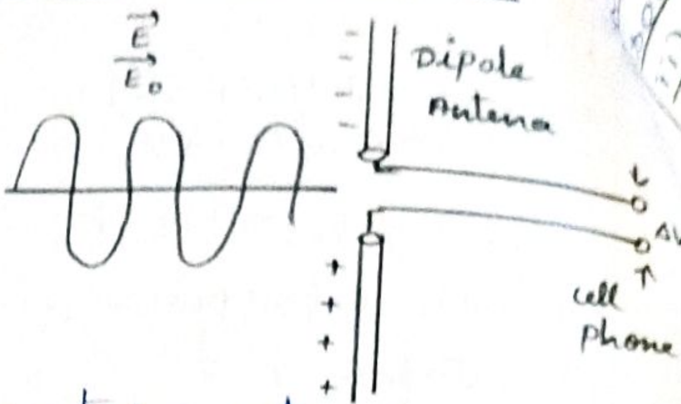
$P_r = F/A$ $P_r = I/c$

Total absorption of radiation

$P_r = 2I/c$

for total reflection back along the Path.

ii) cell phone Reception



- contains a tiny low-Power radio transmitter or antenna
- EM signal intensity decreases as the inverse square of the distance from the phone.
- Antenna's length is comparable to $\lambda/2$, λ - wavelength.
- λ is short cell phone antenna is very short.
- EM signal induces a voltage across the wires of the antenna
- Induced voltage is amplified
- Low Power signals emitted by the cell phone will be received and transmitted by the cell phone towers.
- Towers are another type of antenna
- cell phone transmitts one one frequency and receive with other frequency.

Write short notes in i) Localized sources for Electromagnetic waves ii) Polarization iii) Producing electromagnetic waves.

i) Localized sources for EM waves

Electromagnetic waves can be produced either

- by accelerated electric charges
- by time varying electric currents

Magnetic field vector is mutually perpendicular to both electric field and the direction of wave propagation

$$E_y = E_0 \cos(\omega t - kx)$$

$$B_z = B_0 \cos(\omega t - kx)$$

In free space or vaccum the ratio between E_0 and B_0 is equal to the speed of electromagnetic wave is equal to speed of light c.

$$c = E_0 / B_0$$

$$v = E_0 / B_0 < c$$

The energy of electromagnetic waves comes from the energy of the oscillating charge.

ii) Polarisation

The Phenomenon by which the vibrations of the electric field vector of an electromagnetic wave to a particular plane is called Polarization.

The plane in which the electric field oscillates is defined as the plane of Polarization.

Plane Polarized wave

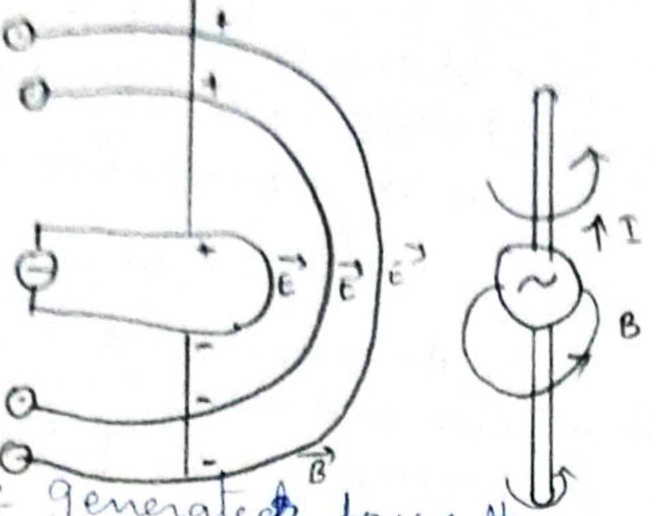
If the variation of electric field is observed along the direction of propagation, tip of electric field vector appears to trace a straight line along vertical direction. In this vibration, E vector is confined to a single plane perpendicular to direction of propagation. such wave is known as plane Polarized wave

Types of Polarization.

i) Linear Polarization.
ii) circular Polarization
iii) Elliptical Polarization.

iii) Producing electromagnetic waves

- steady currents can produce electromagnetic waves.
- EM waves are the combination of electric and magnetic field produced by moving charges.
- consider two conducting ~~charges~~ rods connected to a source of alternating voltage.



- AC generator forces the charges to accelerate between two rods.
- Antenna compared to an oscillating electric dipoles.
- Produced by the charge distribution on the wire
- E and the charge distribution vary as the current changes.
- The changing field propagates outward at the speed of light
- The electric and magnetic fields are 90° out of the phase at all times.
- The electric and magnetic fields are closely related and propagate as an electromagnetic wave.
- Time varying electric field.
- The result is the outward flow of electromagnetic wave energy at all times.

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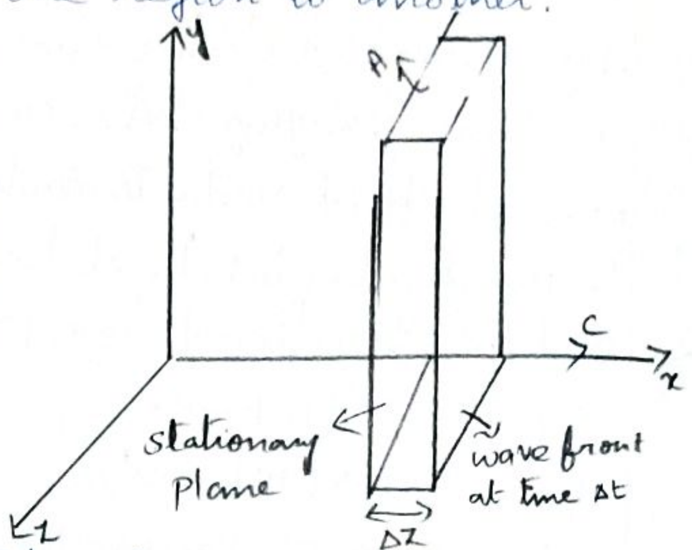
14) Derive an expression for Electromagnetic energy flow & Poynting vector

i) Electromagnetic Energy flow and Poynting vector.

- EM waves transports energy from one region to another.



At a time Δt after this wave front moves a distance to the right side of the plane

$$\Delta x = c\Delta t$$

$$\Delta V = A \cdot \Delta x = A \cdot c \cdot \Delta t$$

If ΔU is the available energy

$$\Delta U = u\,\Delta V = (\varepsilon_0 E^2)(A c \Delta t) \quad \text{—} (1)$$

u energy density is equal to $\varepsilon_0 E^2$

$$S = \frac{\Delta U}{A \cdot \Delta t} = \varepsilon_0 E^2 c \quad \text{——} (2)$$

$E = cB$, $c = 1/\sqrt{\mu_0 \varepsilon_0}$

equ (2) becomes

$$S = \varepsilon_0 c^2 B^2 c \quad \text{——} (3)$$

$$S = \frac{\varepsilon_0 C B^2}{\varepsilon_0 \mu_0} \quad \text{--- (4)}$$

$$S = \frac{CB.B}{\mu_0} = \frac{EB}{\mu_0} \quad \text{--- (5)}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad \text{--- (6)}$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

equ (6) is the Poynting vector in Vacuum.

ii) Intensity of an EM wave in Vaccum

Let us consider electric and magnetic field solution

$$\vec{E}(x,t) = E_y \cos(\omega t - kx) \times B_z \cos(\omega t - kz) \quad \text{--- (9)}$$

$$S_x(x,t) = \frac{E_y B_z}{\mu_0} \cos^2(\omega t - kx)$$

$$= \frac{E_y B_z}{\mu_0} \left(\frac{1 + \cos 2(\omega t - kx)}{2} \right) \quad \text{--- (11)}$$

The time average value of $\cos 2(\omega t - kx)$ is zero. So the average value of the Poynting vector.

$$S_{average} = \vec{S}_x(x,t) = \frac{E_x B_y}{2\mu_0} \quad \text{--- (12)}$$

or simply

$$S_{av} = \frac{E_y B_z}{2\mu_0} = \frac{E_y \cdot E_y}{2\mu_0 c} \quad \text{--- (13)}$$

$$= \frac{E_y \cdot E_y}{2\mu_0 \times \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

$$S_{av} = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} \, E_y^2$$

$$S_{av} = \frac{1}{2} \sqrt{\frac{\varepsilon_0 \varepsilon_0}{\mu_0 \varepsilon_0}} \, E_y^2$$

$$S_{av} = \frac{\varepsilon_0}{\sqrt{\mu_0 \varepsilon_0}} E_y^2$$

$$\text{Intensity } I = S_{av}$$

$$= \frac{1}{2} \varepsilon_0 C E_y^2 \quad \text{--- (14)}$$

This is the intensity of an EM wave in a vaccum Intensity as localised sources as

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi r^2}$$

UNIT - III

Oscillations, optics and Lasers

LASERS

Theory of laser - characteristics - spontaneous and stimulated emission - Einsteins coefficients - Population inversion - Nd-YAG laser, CO_2 laser, semiconductor laser - Basic applications of laser in industry.

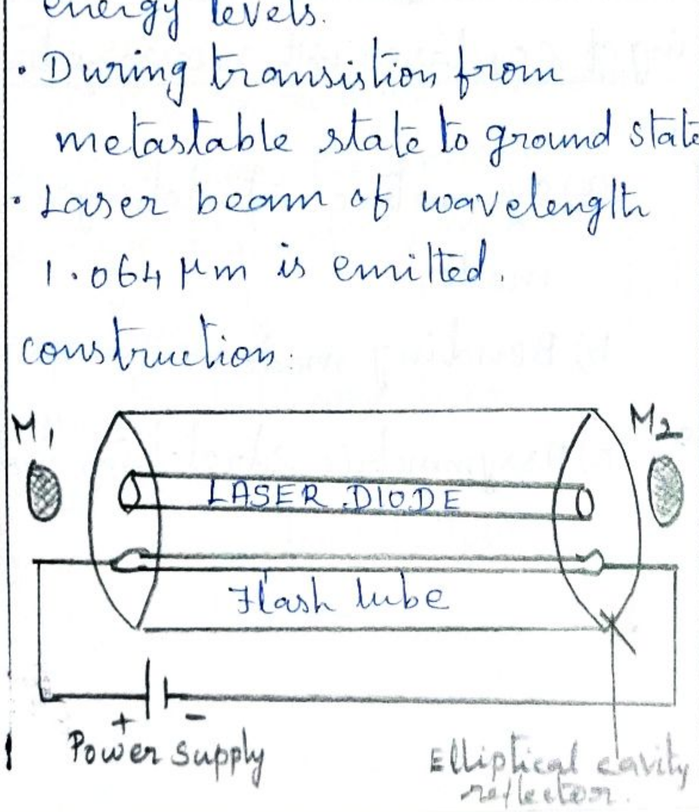
① Explain the construction and working of Nd:YAG Laser with neat diagram

- Nd: YAG - Neodymium based laser [Neodymium Yitrium Aluminium Garnet]
- It is a four level solid state laser.

Principle

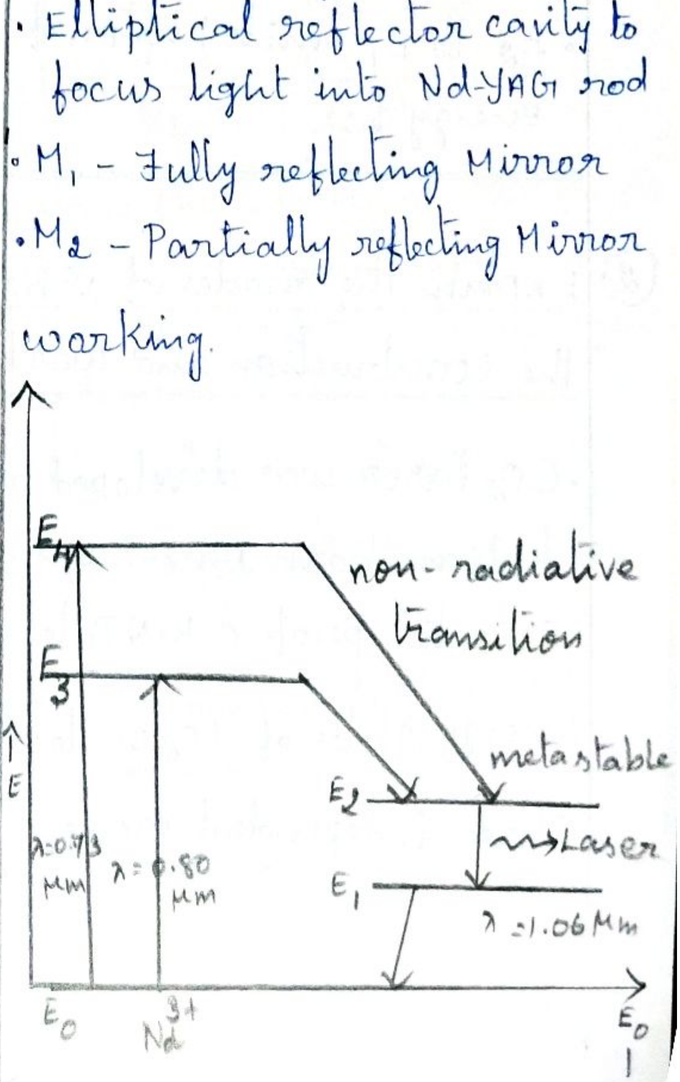
- Active medium Nd:YAG rod is optically pumped by krypton flash tube.
- Nd^{3+} ions are raised to excited energy levels.
- During transistion from metastable state to ground state
- Laser beam of wavelength 1.064 μm is emitted.

construction:



- Ytrium ions replaced with neodymium ions in active medium.
- Ends of rod are highly polised and optically flat and parallel
- Elliptical reflector cavity to focus light into Nd-YAG rod
- M_1 - Fully reflecting Mirror
- M_2 - Partially reflecting Mirror

working.



- When krypton flash tube is switched on, Nd^{3+} ions are excited from ground state E_0 to upper energy levels E_3 and E_4.
- Due to absorption of light radiation of wavelengths 0.73 μm and 0.80 μm.
- Excited energy levels make a transition to energy level E_2 by non-radiative transition
- Neodymium ions are collected in E_2 energy level.
- Population inversion is achieved between E_2 and E_1
- E_2 to E_1 emits photon of energy $h\nu$.
- Emitted photon triggers chain of stimulated photons between E_2 and E_1

characteristics

Type : Four level laser

Active medium : Nd:YAG rod

Pumping method : optical Pumping

Pumping source : Krypton flash tube

Power output : 20 KW.

Advantages

- High energy output
- To achive population inversion

Disadvantages

- more complicated

Applications

- Used in range finders and illuminators
- Medical applications such as endoscopy, Urology, neurosurgery

② Explain the modes of vibrations of CO_2 molecule. Describe the construction and working of CO_2 laser with necessary diagrams

- CO_2 laser was developed by Indian born American scientist prof C.K.N Patel.

Energy states of CO_2 molecule

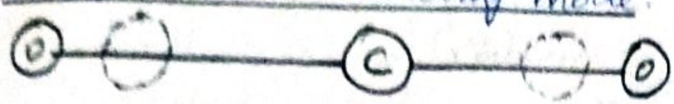
- Three independent mode.

a) Symmetric stretching mode.

b) Bending mode.

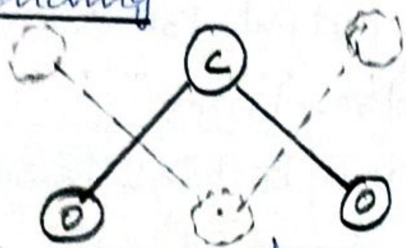
c) Assymmetric stretching mode.

a) Symmetric stretching mode.



- carbon atom is at rest.
- Both oxygen atoms vibrate and moving away.

b) Bending



- Both oxygen atoms and carbon atom vibrate Perpendicular to molecular axis.

c) Asymmetric stretching.

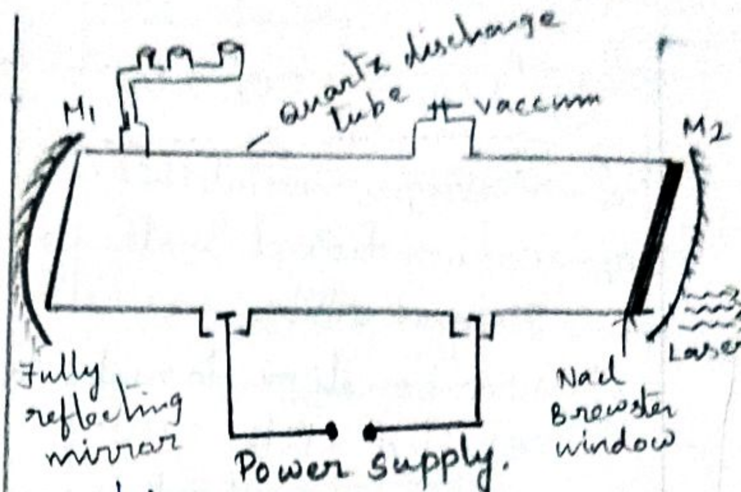
- Both oxygen atoms and carbon atom vibrate asymmetrically.

Principle

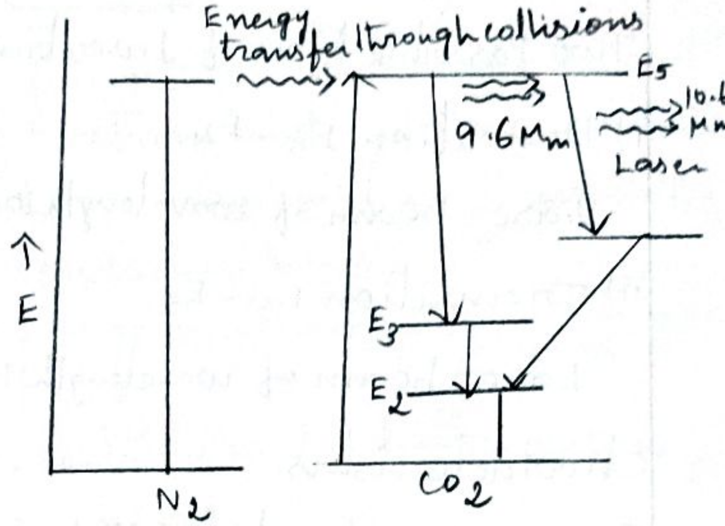
The laser transition takes Place between the vibrational energy states of CO_2 molecules.

construction.

- quartz discharge tube
- H_2, N_2, CO_2
- Nacl Brewster windows
- M_1, M_2 - mirror



warking



- when the electrical discharge occurs in gas mixure, the electrons collide with nitrogen molecules.

$$N_2 + e^* \longrightarrow N_2^* + e.$$

N_2 - Nitrogen molecule in ground state

$e^* \rightarrow$ Electron with high energy state

$N_2^* \rightarrow$ Nitrogen molecule in excited

$e \rightarrow$ Same electron with lesser energy.

$N_2^* + CO_2 \longrightarrow CO_2^* + N_2$

N_2^* → Nitrogen molecule in excited state

CO_2 → Carbon dioxide molecule in ground state.

CO_2^* → carbondioxide molecule in excited state.

N_2 → Nitrogen molecule in ground state.

Two Possible types of Laser transition

i) Transition $E_5 - E_4$

Laser beam of wavelength 10.6 μm

ii) Transition $E_5 - E_3$

Laserbeam of wavelength 9.6 μm

Characteristics

Type : Four level laser

Active medium : CO_2, N_2, He

Pumping Method : electrical discharge method.

Power output : 10 KW.

Advantages

- CO_2 laser is simple
- High efficiency
- High output Power.

Disadvantages

Due to high Power laser light damage eyes.

Applications.

- Used in remote sensing
- Used in treatment of liver and lung diseases
- It is used in neurosurgery and general surgery.

---

③ with suitable diagram explain how laser action is achieved in homojunction Ga-As laser.

Semi conductor diode laser

i) Homojunction Semiconductor Lasers.

- Use same type of semiconductor material.

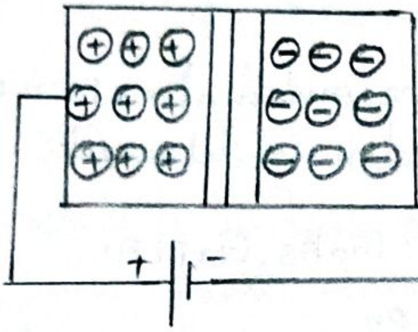
Example : Gallium Arsenide (GaAs)

Definition

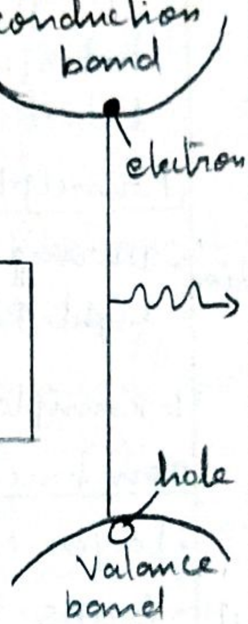
- It is a fabricated P-n junction diode
- Diode emits laser light when it is forward biased

Principle:

- Due to recombination process light radiation (Photons) is released.
- Stimulated emission takes place and laser light is produced.



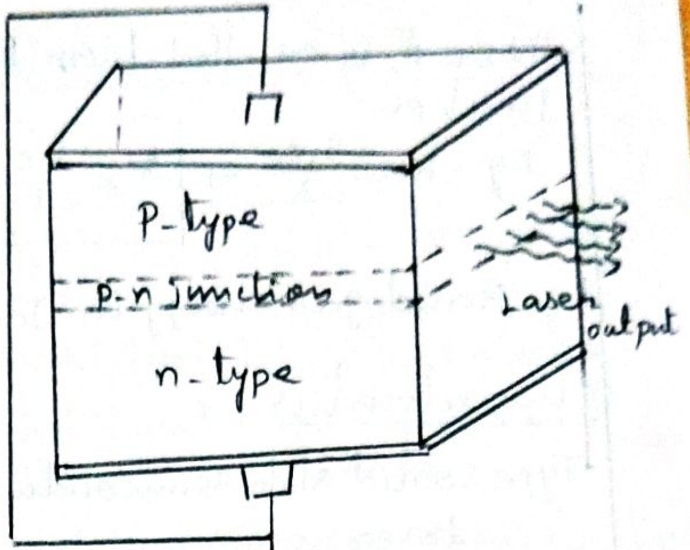
(a)



(b)

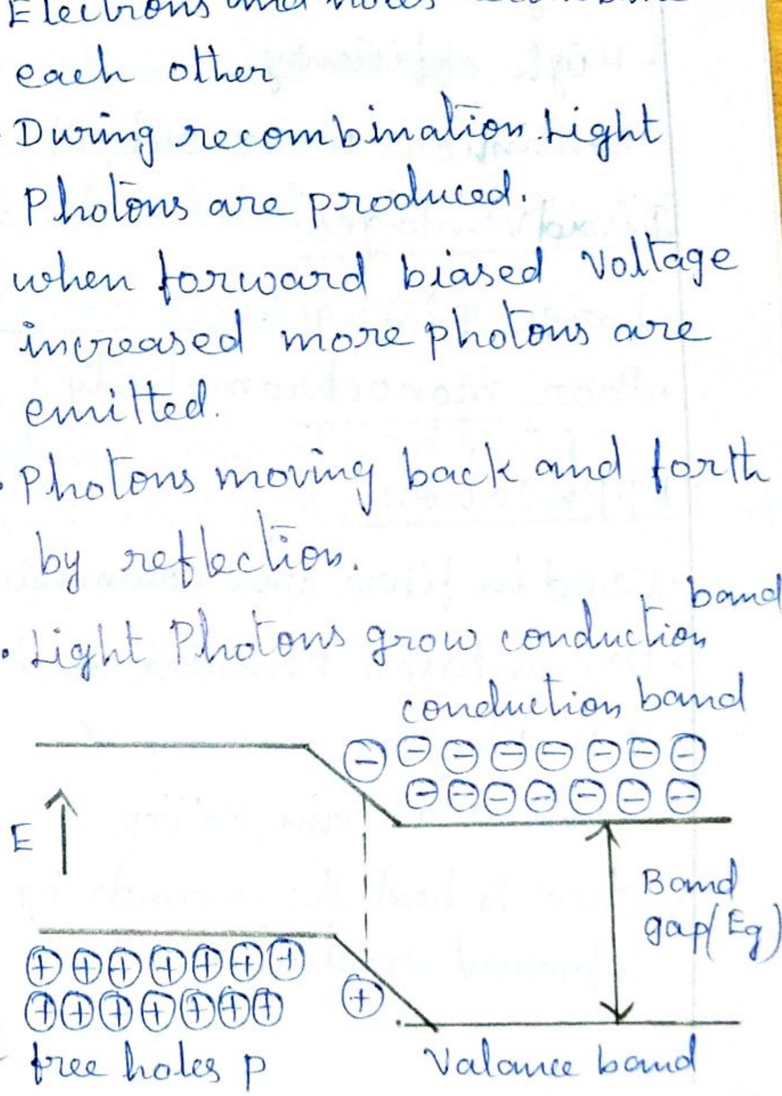
Construction

- Active medium is a P-n junction diode, a single crystal of gallium arsenide.
- Two regions n-type, P-type.
- Electrodes are connected both Upper and lower region.
- Forward bias voltage is applied
- The end faces are well Polished and Parallel.
- Emitted light comes out through the optical resonator.



Working

- P-n junction is forward biased
- Electrons and holes are injected.
- Large number of electrons in the conduction band.
- Large number of holes in valance band.
- Electrons and holes recombine each other
- During recombination Light Photons are produced.
- when forward biased voltage increased more photons are emitted.
- Photons moving back and forth by reflection.
- Light Photons grow conduction band



• Photon is emitted from the Junction

$$E_g = h\nu = \frac{hc}{\lambda} \Rightarrow \boxed{\lambda = \frac{hc}{E_g}}$$

E_g - Band gap energy in Joule.

Characteristics

• Type : Solid state Semiconductor laser.

• Active medium : GaAs.

• Pumping Method : Direct conversion method

• Wavelength of output : 8300Å to 8500Å

Advantages

• Very small in size and compact

• High efficiency

• continuous wave output.

Disadvantages:

• Large Divergence.

• Poor monochromaticity.

Application.

• Used in fibre optic communication

• Use in laser printers and CD players.

• Used as a Pain Killer.

• Used to heal the wounds by infrared radiation.

ii) Hetrojunction Semiconductor laser.

Definition.

• A diode laser with a Pn Junction made up of different semiconductor materials in two regions.

• n-type and P-type is known hetrojunction semiconductor laser.

Principle:

• During recombination process, Light Photon is released.

Example : GaAs, GaAlAs.

construction

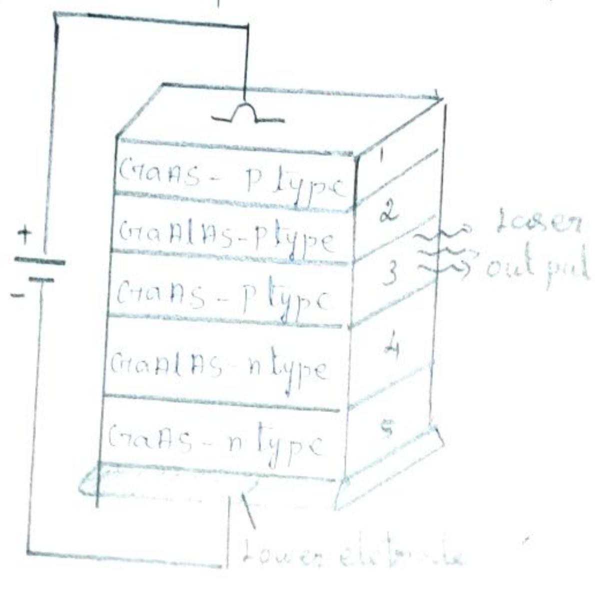
• Layer consists of the layers.

• GaAs P-type in third layer as active region.

• GaAlAs - Ptype (2nd layer)
GaAlAs - ntype (4th layer)

• 3rd and 4th layers are well Polished and Parallel

• Act as optical resonator



List out the applications of laser beam in industries and in medical field

Industrial Applications

- Material processing like cutting drilling and welding.
- Lasers are used to scan the universal barcodes to identify products.
- Used to take 3-D photography
- In the field of chemistry, used to initiate chemical and Photo chemical reactions.
- In the field of fibre optic communication, thousands of television programs and telephone conversations can be transmitted simultaneously using laser beam
- To store and retrieve data in optical discs.

Medical Applications

- In the treatment of detached retinas.
- To perform micro surgery and bloodless operations. to cure cancers and skin tumors.
- Nose, ear, throat surgery
- To remove Kidney stones.

other applications.

- As range finder in military application - LIDAR. (Light detection And Ranging)
- Under water communication between submarines.
- To determine ozone concentration
- To measure the distance between earth and moon accurately.

5 Derive Einstein relations (A & B)

- consider an atom
- when light radiation incident on atoms, three different processes takes place.

a) stimulated absorption.
b) Spontaneous emission
c) stimulated emission.

a) Stimulated absorption

- Atoms in the lower energy state E_1 absorbs radiation and is excited to the higher energy level E_2. This process called stimulated or induced absorption.

$$N_{ab} \propto N_1 Q$$

$$N_{ab} = B_{12} N_1 Q \quad \text{——— (1)}$$

B_{12} - Proportionality constant

- This process is an upward transition.

b) Spontaneous emission

- Atoms in the excited state E_2 return to lower energy state E_1 Emitting photon of energy $h\nu$. This emission of light radiation is known as spontaneous emission

$$N_{sp} \propto N_B$$

The number of transition per second is given by.

$$N_{sp} = A_{21} N_2 \quad \text{——} (2)$$

A_{21} - proportionality constant

- This process is a downward transition.

c) Stimulated emission

- Light photon is incident on the atom in the excited energy state, the photon triggers the excited atom to make transition to lower energy E_1

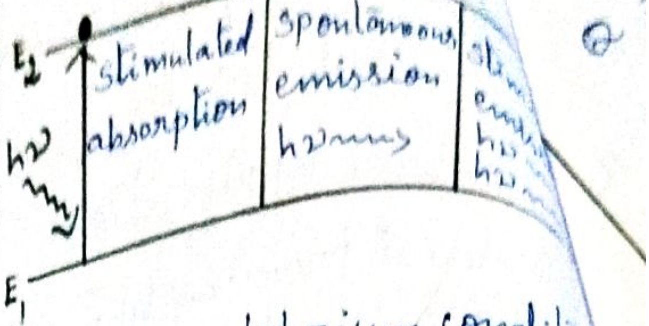
$$N_{st} \propto N_2 Q.$$

The number of transitions per second.

$$N_{st} = B_{21} N_2 Q \quad \text{——} (3)$$

B_{21} - Proportionality constant

- This process is a downward transition. The proportionality constants A_{21}, B_{12} and B_{21} are known as Einstein's coefficients A & B.



- Under equilibrium condition the number of downward and upward transitions are equal

$$N_{sp} + N_{st} = N_{ab} \quad \text{——} (4)$$

Substituting equs (1) (2) (3) in equ (4)

$$A_{21} N_2 + B_{21} N_2 Q = B_{12} N_1 Q \quad \text{——} (5)$$

Rearranging the equ (5)

$$B_{12} N_1 Q = B_{21} N_2 Q = A_{21} N_2$$

$$Q(B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$Q = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} \quad \text{——} (6)$$

Dividing numerators and denominator by $B_{21} N_2$

$$Q = \frac{\dfrac{A_{21} N_2}{B_{21} N_2}}{\dfrac{B_{12} N_1}{B_{21} N_2} - \dfrac{B_{21} N_2}{B_{21} N_2}}$$

$$Q = \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{B_{12}}{B_{21}}\right) \frac{N_1}{N_2} - 1} \quad \text{——} (7)$$

on substituting $\frac{N_1}{N_2} = e^{h\nu/kT}$ in equ (7)

$$Q = \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{B_{12}}{B_{21}}\right) e^{h\nu/kT} - 1} \quad \text{—— (8)}$$

Planck's radiation formula for energy distribution is given by

$$Q = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} \quad \text{—— (9)}$$

comparing the eqns (8) & (9)

$$\frac{B_{12}}{B_{21}} = 1 \quad \text{—— (10)}$$

$$B_{12} = B_{21}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \text{ (or) } = \frac{8\pi h}{\lambda^3} \quad \text{—— (11)}$$

$B_{12} = B_{21}$. Einstein's coefficients are termed as A and B coefficients.

Optics

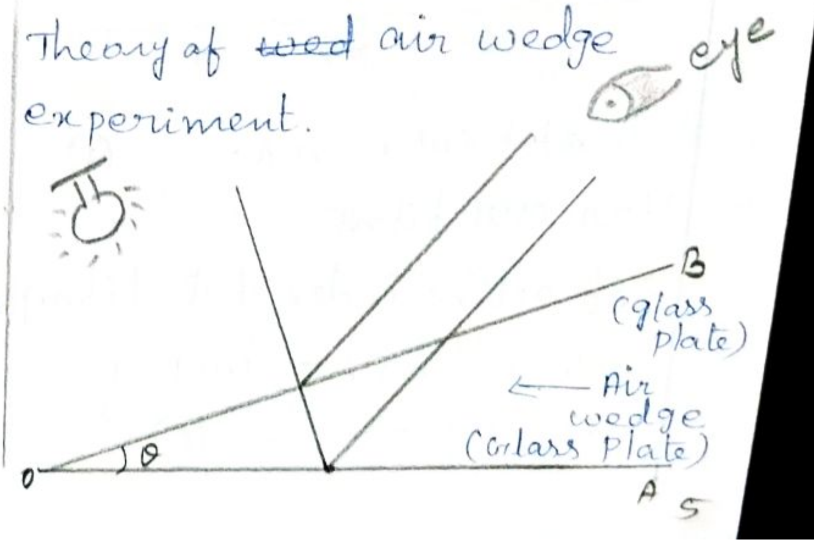
Reflection and refraction of light waves - total internal reflection - interference Michelson interferometer - Theory of airwedge and experiment.

Explain the formation of interference fringes in an air wedge shaped film. How is the thickness of the wire determined by this method.

Definition.

A wedge shaped (V-shaped) air film enclosed in between two flat glass plates is called airwedge.

Theory of ~~wed~~ air wedge experiment.

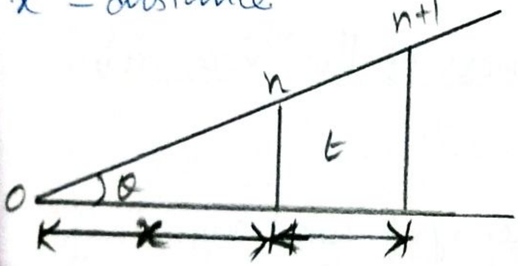


UNIT-IV

- when two optically plane glass plates (A and B) are inclined at a very small angle θ, a wedge shaped thin air film is formed.
- light rays from a monochromatic light source is made to fall Perpendicularly on the film
- Incident light rays.
- Partially reflected - Upper surface
- Partially reflected - lower surface.
- Two reflected rays interfere and a large number of straight alternative bright and dark fringes are formed.
- t - thickness of the air film

θ - angle

x - distance



$2\mu t \cos r = n\lambda$ —— (1)

For air film

refractive index of the film $\mu = 1$

$\cos r = 1$ ie) $r = 0$, $\cos 0 = 1$

$2t = n\lambda$ —— (2)

λ - wavelength of light

since x is the distance of the n^{th} dark band from the edge of contact O.

$\frac{t}{x} = \tan\theta$

$\frac{t}{x} = \theta$ (θ is small $\tan\theta \approx \theta$)

$t = x\theta$ —— (3)

Substituting equ (3) in equ (2)

$2x\theta = n\lambda$ —— (4)

for the next dark band

ie) $(n+1)^{th}$ dark band.

$2(x+\beta) = (n+1)\lambda$ —— (5)

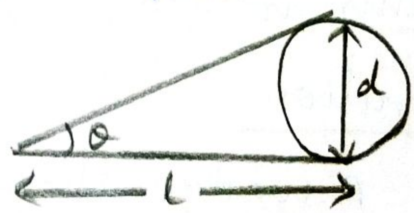
β - fringe width.

subtracting equ (4) from equ (5)

$2\beta\theta = \lambda$

$$\boxed{\beta = \frac{\lambda}{2\theta}} \text{ —— (6)}$$

Thickness of a thin wire and very thin foil.



thickness - d, distance l

$\tan\theta = \frac{d}{l}$ ($\therefore \tan\theta \approx \theta$)

$\theta = \frac{d}{\lambda}$ —— (7)

substituting equ (7) in (6)

$$\beta = \frac{\lambda}{\frac{2d}{l}} = \frac{\lambda l}{2d}$$

$$\boxed{d = \frac{\lambda l}{2\beta}}$$

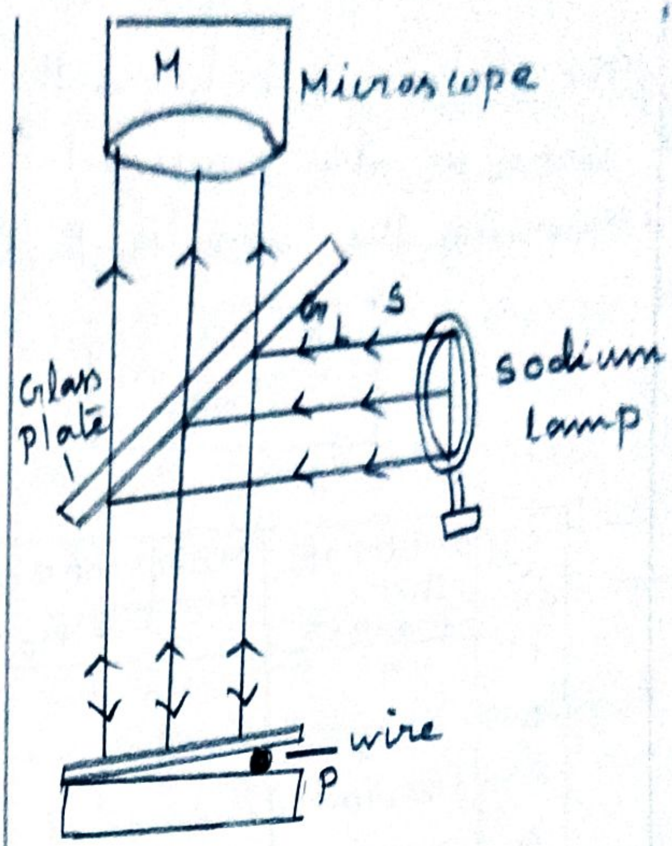
Applications of airwedge

Determination of diameter of a wire (or) Thickness of a thin sheet of paper (Experiment)

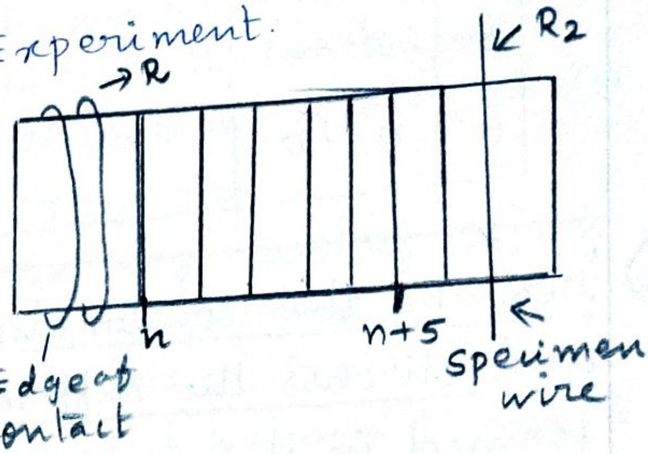
- Airwedge is formed by keeping two optically plane glass plates in contact along one of the edges of a thin wire and the other end parallel to the contact edges of the glass plates
- glass plates are inclined at a very small angle θ. This is called airwedge arrangement

Description.

- Microscope
- Sodium lamp
- Glass plate
- wire
- Glass plate kept inclined at an angle 45° to the horizontal



Experiment.



Interference pattern consisting a series of bright and dark bands equal width.

- Reading noted.
- Cross wire coincide with successive 5th fringes (n+5, n+10, ... n+40) and the readings are noted.
- The readings are recorded.
- The average fringe width β is determined

- The distance l between the edge of the contact and the wire is also measured.
- Knowing the wavelength, thickness of the wire is found.

$$\boxed{d = \frac{l\lambda}{2\beta}\ m}$$

S.No	order of the fringes	Microscope reading $\times 10^{-2}$ m	width of 10 fringes m	Band width β m
1	n			
2	n+5			
3	n+10			
4	n+20			
5	n+30			
6	n+40			

(2) i) Describe the construction of a Michelson's Interferometer and discuss the different types of interference fringes formed in it.

ii) How will you use it to determine the wavelength of a monochromatic source?

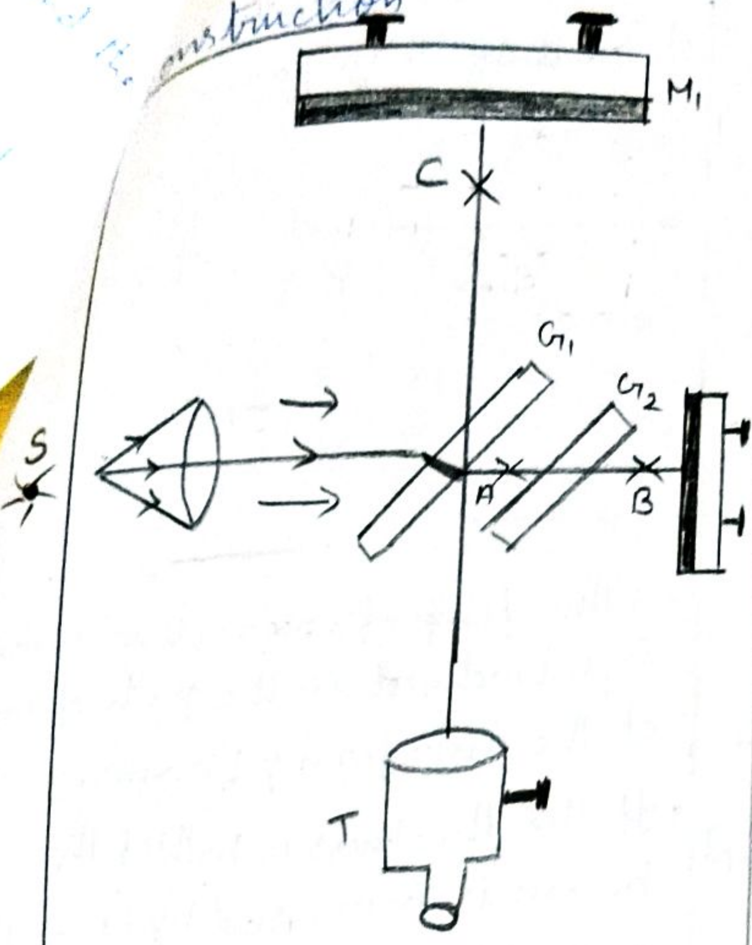
Interferometer

An interferometer is an instrument for measuring small changes in length.

Principle

- Two interfering beams are formed by splitting the light from a source into two parts by partial reflection and refraction.
- These beams sent in two perpendicular directions.
- After reflection from plane mirrors to produce interference fringes.

Construction



- M_1, M_2 – Highly Polished Plane Mirrors
- C – carriage
- G_1, G_2 – two plane Parallel glass plates
- T – Telescope.

working

- Light source S parallel by means of a collimating lens L
- Light falls on semi silvered glass plate G_1.
- Lightbeam divided into two parts
- one part of the light is reflected towards mirror M_1
- other part of the ~~mirror~~ light is transmitted towards M_2.
- Light reflected by Mirror M_1 Passes through G_1, to reach the telescope T.
- The ray reflected by Mirrors M_2 on reaching G_1, reflected at its semi silvered Surface to reach the telescope.
- A path difference introduced between the two reflected rays by moving mirror M_1.
- M_1 directly together with a Virtual image of M_2 denoted by M_2'
- The rays reaching the telescope appear to travel from M_1 and M_2'
- Interference from an air film enclosed between M_1 and M_2'
- The interference fringes be straight, circular or parabolic

Formation of fringes.

For maximum intensity in the fringes.

$$\boxed{2d\cos\theta + \lambda/2 = n\lambda}$$

where $n = 0, 1, 2 \ldots$

Types of Fringes

Case (i):

- when M_2' coincides with M_1
- Path difference is $\lambda/2$
- View perfectly dark.

Case (ii)

- M_1 is moved either forward or backward parallel to itself
- spaced circular fringes.

Case (iii)

- when mirror M_1 intersects the virtual image M_2'
- Air film enclosed wedge shaped and straight line fringes are produced.

wavelength determination

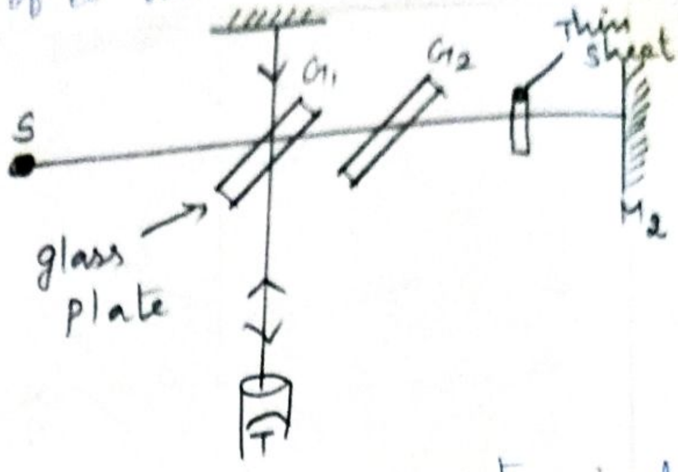
For the shift of n fringes distanced moved by Mirror

$$M_1 = d = \frac{n\lambda}{2}$$

$$\boxed{\lambda = \frac{2d}{n}}$$

- Knowing d and n, the wavelength of monochromatic light λ can be calculated.

Determination of thickness of a thin transparent sheet



- A thin film of refractive index μ is introduced in the path of one of the interfering beams.
- If the thickness t, Path of the beam is increased by $(\mu - 1)$
- Path difference between the beams $2(\mu-1)t$

$$2(\mu - 1)t = n\lambda$$

$$\boxed{t = \frac{n\lambda}{2(\mu-1)}}$$

- If μ, n, λ are known t can be calculated.

Applications.

It is used to find

i) The wavelength of the given light source.

ii) The resolution of wavelengths

iii) The standardisation of metre.

iv) The refractive index and thickness of a transparent material.

Oscillations

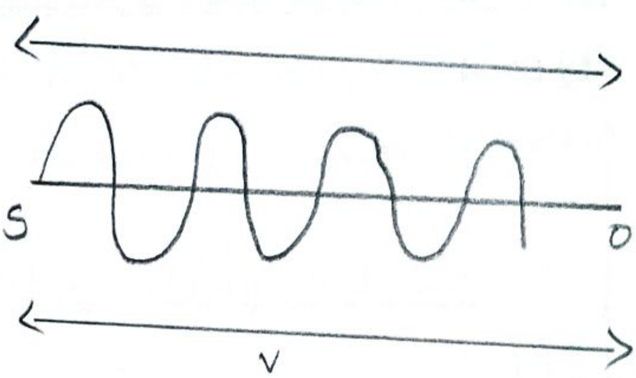
Simple harmonic motion - resonance - analogy between electrical and mechanical oscillating systems - waves on a string - standing waves - travelling waves - Energy transfer of a wave - Sound waves - Doppler effect.

1. State and explain Doppler's effect. Derive an expression for the change in frequency of a note when both the source of sound and the observer are in motion.

Definition

The phenomenon of the apparent change in the frequency of the sound due to relative motion between the source of sound and the observer is called doppler effect.

i) Both source and observer at rest.



- S - Source
- O - Observer
- n - frequency
- V - Velocity of sound

In one second, a waves produce by the source.

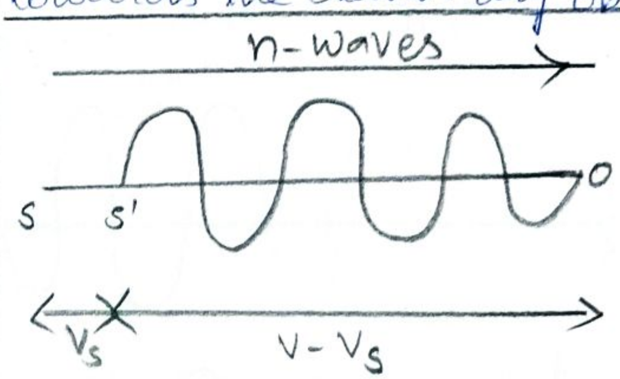
The original wavelength is

$$\lambda = \frac{v}{n} \quad \text{——} \quad (1)$$

The original frequency.

$$n = \frac{v}{\lambda} \quad \text{——} \quad (2)$$

ii) when the source moves towards the stationary observer



$SS' = V_S$

The apparent wavelength of the sound.

$$\lambda' = \frac{v - V_s}{n} \quad \text{——} \; (3)$$

The apparent frequency

$$n' = \frac{v}{\lambda'} = \left(\frac{v}{v - V_s}\right) n \quad \text{——} \; (4)$$

iii) when the sources moves away from the stationary observer

The apparent frequency

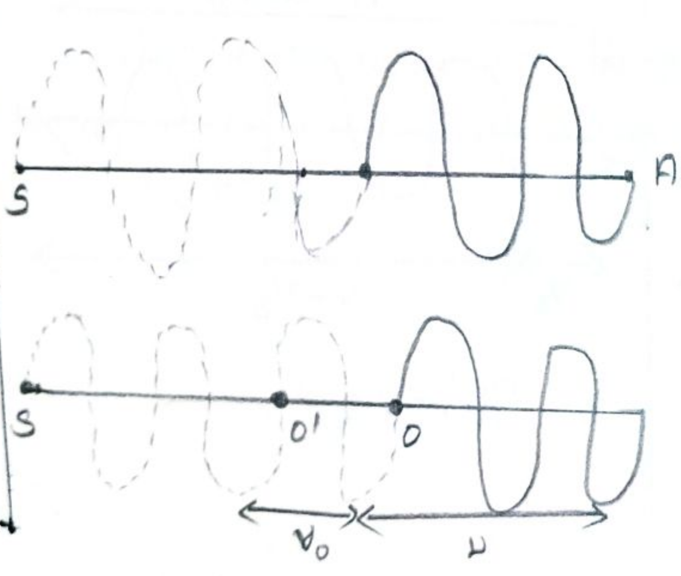
$$n' = \frac{v}{\lambda'} = \left(\frac{v}{v - (-v_s)}\right) n$$

$$= \left(\frac{v}{v + V_s}\right) n \quad \text{——} \; (5)$$

iv) Source is at rest and observer in motion.

- S – Source

$$\lambda = \frac{v}{n}$$



v) when the observer moves towards the stationary source

$$OO' = v_0$$

The apparent frequency of sound

$$n' = n + \frac{v_0}{\lambda} = n + \left(\frac{v_0}{v/n}\right)$$

$$n' = n + \left(\frac{v_0}{v}\right) n$$

$$n' = \left(1 + \frac{v_0}{v}\right) n$$

$$n' = \left(\frac{v + v_0}{v}\right) n \quad \text{——} \; (6)$$

vi) when the observer moves away from the stationary source

The apparent frequency of sound

$$n' = \left(\frac{v + (-v_0)}{v}\right) n$$

$$n' = \left(\frac{v - v_0}{v}\right) n \quad \text{——} \; (7)$$

~~2) wari~~

Write the applications of Doppler effect.

i) To measure the speed of an automobile

- Electromagnetic wave emitted by a source (Police car)
- Shift in frequency of the reflected wave.

ii) RADAR (Radio detection and ranging)

- High frequency radio waves towards an aeroplane.
- reflected waves are detected by the receiver of the radar station.
- Used to determine the speed of an aeroplane.

iii) SONAR (Sound navigation and ranging)

- Sound waves generated from a ship fitted with SONAR are transmitted in water towards an approaching Submarine.
- The frequency of the reflected waves is measured.
- The speed of the submarine is calculated.

iv) Blood flowmeter.

- Ultrasonic sounds are transmitted towards organs.
- frequency change in reflected waves.

v) Tracking satellite

- The frequency received by the Earth station combined with the constant frequency generated in the station gives the beat frequency.

UNIT - IV

Basic Quantum Mechanics

Photons and light waves - Electrons and matterwaves - compton effect - The schrodinger equation (Time dependent and time independent forms) meaning of wave function - Normalization - free particle - Particle in a infinite Potential well: 1D, 2D, 3D Boxes - Normalization, Probabilities and the correspondence principle.

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(1) **Explain about the concept of Matterwaves (Electrons and Matter waves)**

- In 1924 De Broglie extended the idea of dual nature of radiation to matter.
- Motion of electron with in an atom is guided by a peculiar Kind of waves called 'pilot waves'.

De Broglie Hypothesis

- Dual characteristics
- Particle-like, wave-like.

De-Broglie waves and its wavelength.

The waves associated with the matter Particles are called matter waves or debroglie waves.

From planck's theory

$$E = h\nu \quad \text{——} (1)$$

According to Einstein's mass energy relation

$$E = mc^2 \quad \text{——} (2)$$

m - mass of the Photon

c - Velocity of the photon.

Equating (1) and (2)

$$h\nu = mc^2 \quad \text{——} (3)$$

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{hc}{mc^2}$$

$$\lambda = \frac{h}{mc}$$

$mc = P$ momentum of a Photon

$$\lambda = h/P \quad \text{——} (4)$$

UNIT-V

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{——— (5)}$$

equ (5) is known as de-Broglie's wave equation.

De-Broglie wavelength in terms of energy.

$$K.E = \frac{1}{2} m v^2$$

multiplying by m on both sides

$$mE = \frac{1}{2} m^2 v^2 \quad \text{——— (6)}$$

$$2mE = m^2 v^2$$

$$m^2 v^2 = 2mE$$

Taking square root on both sides

$$\sqrt{m^2 v^2} = \sqrt{2mE}$$

$$mv = \sqrt{2mE}$$

$$\lambda = \frac{h}{mv} \quad \text{——— (7)}$$

Substituting mv value on equ (7)

$$\lambda = \frac{h}{\sqrt{2mE}}$$

De-Broglie wavelength in terms of accelerating Potential associated with electrons.

workdone on the electron

$$= eV \quad \text{——— (1)}$$

workdone = K.E

$$eV = \frac{1}{2} m v^2 \quad \text{——— (2)}$$

$$2eV = mv^2$$

$$mv^2 = 2eV$$

xly by m on both sides

$$m^2 v^2 = 2meV$$

Taking square root on both sides

$$\sqrt{m^2 v^2} = \sqrt{2meV}$$

$$mv = \sqrt{2meV} \quad \text{——— (3)}$$

$$\lambda = \frac{h}{mv} \quad \text{——— (4)}$$

Substituting equ (3) in equ (4)

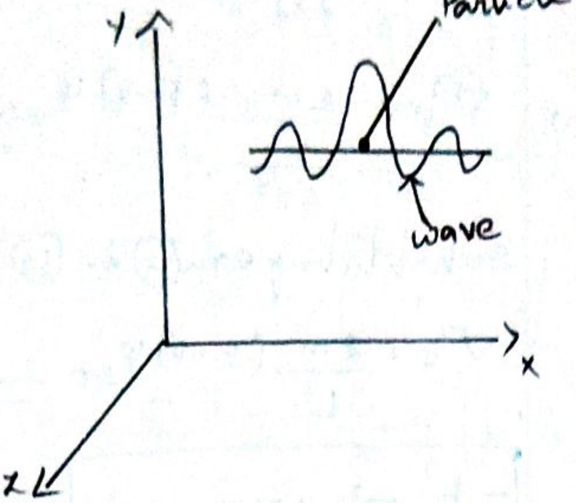
$$\boxed{\lambda = \frac{h}{\sqrt{2meV}}} \quad \text{——— (5)}$$

Properties of Matter waves.

- If mass of the Particle is smaller then wavelength is longer.
- If velocity is small, wavelength is longer.
- The velocity of de-broglie waves is not constant, it depends on the velocity of the material Particle.

Derive an equation for schrodinger Time independent and dependent wave equation.

i) Schrodinger Time independent equation



- x, y, z - coordinates
- ψ - wave function
- t - time.

The classical differential equation for wave motion.

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad (1)$$

v - wave velocity.

equ (1) is written as

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad (2)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

is the Laplacian's operation

The solution of equ (2) gives ψ as a Periodic variations interms of time t.

$$\psi(x,y,z,t) = \psi_0(x,y,z)e^{-i\omega t}$$

$$\psi = \psi_0 e^{-i\omega t} \quad (3)$$

Differentiating the equ (3) with t,

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t}$$

Again differentiating with respect to t

$$\frac{\partial^2\psi}{\partial t^2} = (-i\omega)(-i\omega)\psi_0 e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = i^2\omega^2\psi_0 e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi \quad (4)$$

$$\left[\because i^2 = -1,\ \psi = \psi_0 e^{-i\omega t}\right]$$

Substituting equ (4) in equ (2)

$$\nabla^2\psi = \frac{-\omega^2}{v^2}\psi$$

$$\nabla^2\psi + \frac{\omega^2}{v^2}\psi = 0 \quad (5)$$

$$\omega = 2\pi\nu = 2\pi\left(\frac{v}{\lambda}\right)$$

ν - frequency $\quad \left(\nu = \frac{v}{\lambda}\right)$

$$\frac{\omega}{v} = \frac{2\pi}{\lambda} \quad (6)$$

Substituting equ (6) on both sides

$$\frac{\omega^2}{v^2} = \frac{2^2\pi^2}{\lambda^2} = \frac{4\pi^2}{\lambda^2} \quad (7)$$

Substituting equ (7) in equ (5)

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \quad (8)$$

$\lambda = \frac{h}{mv}$ → in equ (8)

$$\nabla^2\psi + \frac{4\pi^2}{\frac{h^2}{m^2v^2}}\psi = 0$$

$$\nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \quad \text{——— (9)}$$

Total Energy = Potential energy + K.E

$E = V + \frac{1}{2}mv^2$

$E - V = \frac{1}{2}mv^2$

$2(E-V) = mv^2$

$mv^2 = 2(E-V)$

xlying by m on both sides

$m^2v^2 = 2m(E-V)$ ——— (10)

Substituting equ (10) in equ (9)

$$\nabla^2\psi + \frac{4\pi^2}{h^2} \times 2m(E-V)\psi = 0$$

$$\boxed{\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0} \quad \text{——— (11)}$$

equ (11) is Known as schrodinger time independent wave equ for three dimensions.

$\hbar = \frac{h}{2\pi}$ in equ (11)

$$\hbar^2 = \frac{h^2}{2^2\pi^2} = \frac{h^2}{4\pi^2} \quad \text{——— (12)}$$

$\hbar$ - reduced planck's constant.

equ (11) is modified by 2

$$\nabla^2\psi + \frac{m}{\frac{h^2}{8\pi^2}}(E-V)\psi = 0$$

$$\nabla^2\psi + \frac{m}{\frac{h^2}{2\times2^2\pi^2}}(E-V)\psi = 0$$

$$\nabla^2\psi + \frac{2m}{\frac{h^2}{4\pi^2}}(E-V)\psi = 0 \quad \text{——— (13)}$$

Substituting equ (12) in (13)

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{——— (14)}$$

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi}$$

Special case.

- One dimentional motion, Particle moving along only x-directions equ (14) reduces to

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

Schrodinger Time dependent wave equation

The solution of classical differential equ of wave motion is given by.

$$\psi(x,y,z,t) = \psi_0(x,y,z)e^{-i\omega t} \quad \text{——— (1)}$$

Differentiating equ(1) with t

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t} \quad \text{——— (2)}$$

$$\frac{\partial \psi}{\partial t} = -i(2\pi\nu)\, \psi_0 e^{-i\omega t}$$

$(\because \omega = 2\pi\nu)$

$$\frac{\partial \psi}{\partial t} = -2\pi i \nu \psi \quad \text{——— (3)}$$

$(\because \psi = \psi_0 e^{-i\omega t})$

$$\frac{\partial \psi}{\partial t} = -2\pi i \frac{E}{h} \psi \quad \left[E = h\nu \text{ or } \nu = E/h \right]$$

$$\frac{\partial \psi}{\partial t} = -i \frac{E}{\frac{h}{2\pi}} \psi = -i \frac{E}{\hbar} \psi \quad \left[\hbar = \frac{h}{2\pi} \right]$$

$$\frac{\partial \psi}{\partial t} = -i \frac{E}{\hbar} \psi \quad \text{——— (4)}$$

xlying i on both sides in equ(4)

$$i \frac{\partial \psi}{\partial t} = -i \times i \left(\frac{E}{\hbar}\right) \psi = -i^2 \left(\frac{E}{\hbar}\right) \psi$$

$$i \frac{\partial \psi}{\partial t} = \frac{E}{\hbar} \psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi \quad \text{——— (5)}$$

Schrodinger time independent wave equation.

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi = E\psi$$

Substituting for $E\psi$ from equ (5)

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\left(\frac{-\hbar^2}{2m} \nabla^2 + V\right) \psi = i\hbar \frac{\partial \psi}{\partial t} \quad \text{——— (7)}$$

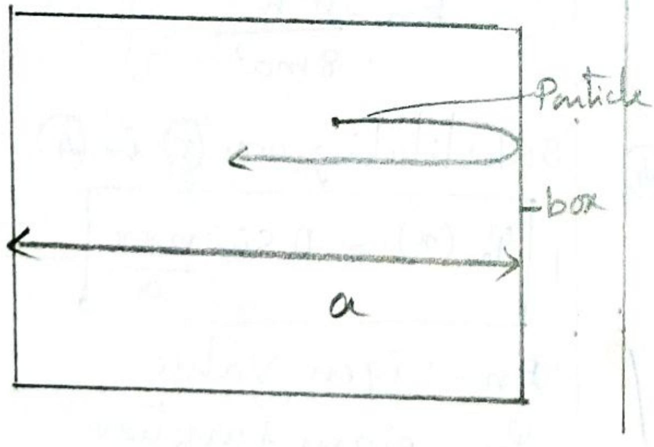
$$\boxed{H\psi = E\psi} \quad \text{——— (8)}$$

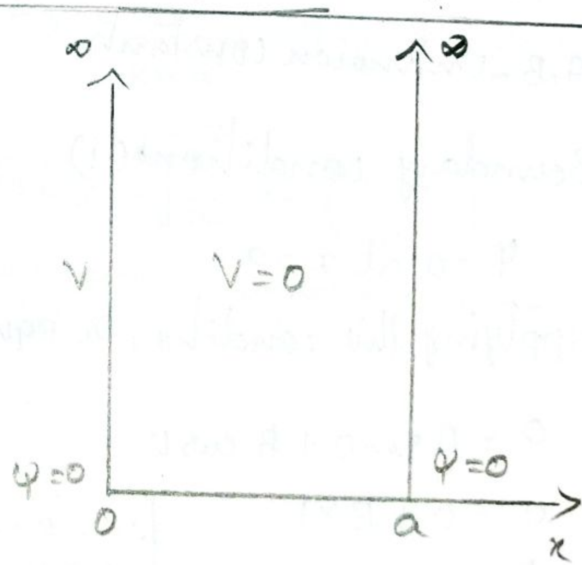
$$H = \left(\frac{-\hbar^2}{2m} \nabla^2 + V\right)$$

Hamiltonean operator

$E = i\hbar \frac{\partial}{\partial t}$ - Energy operator.

3) Solve schrodinger wave eqution of a Particle in box (1D) and obtain the energy eigen values.





- consider a particle of mass m
- $x=0$ and $x=a$

Potential function is given by

$$\boxed{\begin{array}{l} V(x) = 0 \text{ for } 0 < x < a \\ V(x) = \infty \text{ for } 0 \geq x \geq a \end{array}}$$

This potential function is known as square well potential

Schrodinger's wave equ in one dimension.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{—— (1)}$$

V=0 between the walls, equ (1) reduces to

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad \text{—— (2)}$$

Substituting $\frac{2mE}{\hbar^2} = K^2$ in equ (2)

$$\boxed{\frac{d^2\psi}{dx^2} + K^2\psi = 0} \quad \text{—— (3)}$$

The general solution of equ (3)

$$\psi(x) = A\sin Kx + B\cos Kx \quad \text{—— (4)}$$

A, B - Unknown constants.

Boundary conditions (i)

$\psi = 0$ at $x = a$

Applying this condition to equ (4)

$$0 = A\sin 0 + B\cos 0$$

$$0 = 0 + B \times 1 \qquad \boxed{\begin{array}{l} \therefore \sin 0 = 0 \\ \cos 0 = 1 \end{array}}$$

$$B = 0$$

Boundary condition (ii)

Applying this condition to eqn

$$0 = A\sin Ka + 0$$

$$A\sin Ka = 0$$

$$A = 0 \text{ or } \sin Ka = 0$$

$$\sin Ka = 0$$

$$Ka = n\pi$$

$$K = \frac{n\pi}{a} \quad \text{—— (5)}$$

on squaring equ (5)

$$K^2 = \frac{n^2\pi^2}{a^2} \quad \text{—— (6)}$$

$$K^2 = \frac{2mE}{\hbar^2} = \frac{2mE}{\frac{h^2}{4\pi^2}} \qquad \left[\hbar = \frac{h}{2\pi}\right]$$

$$K^2 = \frac{(2m \times 4\pi^2)E}{h^2}$$

$$K^2 = \frac{8\pi^2 mE}{h^2} \quad \text{—— (7)}$$

Equating equ (6) and (7)

$$\frac{n^2\pi^2}{a^2} = \frac{8\pi^2 mE}{h^2}$$

Energy of the Particle

$$E_n = \frac{n^2h^2}{8ma^2}$$

Substituting equ (5) in (4)

$$\boxed{\psi_n(x) = A\sin\frac{n\pi x}{a}} \quad \text{—— (9)}$$

E_n - Eigen value

ψ_n = eigen function.

Normalisation of wave function

Probability density $\psi^* \psi$

$$\psi_n(x) = A \sin \frac{n\pi x}{a}$$

$$\psi^* \psi = A \sin \frac{n\pi x}{a} \times A \sin \frac{n\pi x}{a}$$

$$(\psi * \psi^*) = A^2 \sin^2 \left[\frac{n\pi x}{a}\right] \quad \text{--- (10)}$$

$$\int_0^a \psi^* \psi \, dx = 1 \quad \text{--- (11)}$$

Substituting $\psi^*\psi$ equ (10) in (11)

$$\int_0^a A^2 \sin^2 \left(\frac{n\pi x}{a}\right) dx = 1$$

$$A^2 \int_0^a \frac{\left(1 - \cos\left(\frac{2\pi n x}{a}\right)\right)}{2} dx = 1$$

$$\left[\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$\frac{A^2}{2} \left[\int_0^a dx - \int_0^a \cos\left(\frac{2n\pi x}{a}\right) dx\right] = 1$$

$$\frac{A^2}{2} \left[[x]_0^a - \left(\frac{\sin\left(\frac{2n\pi x}{a}\right)}{\frac{2n\pi}{a}}\right) \right] = 1$$

$$\frac{A^2}{2} [x]_0^a = 1$$

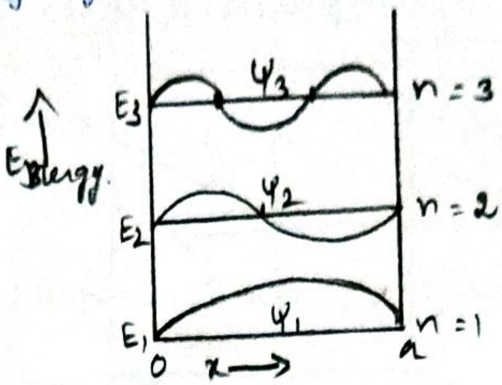
$$\frac{A^2 a}{2} = 1 \quad \text{(or)} \quad A^2 = \frac{2}{a}$$

$$A = \sqrt{2/a}$$

on substituting equ (12) in equ (9)

$$\boxed{\psi_n = \sqrt{2/a} \, \sin \frac{n\pi x}{a}} \quad \text{--- 13}$$

The equ (13) is known as normalised eign function.



Special cases.

case (i) For n = 1

$$E_1 = \frac{h^2}{8ma^2}$$

$$\psi_1(x) = \sqrt{2/a} \, \sin\left(\frac{\pi x}{a}\right)$$

case (ii) For n = 2

$$E_2 = \frac{4h^2}{8ma^2} = 4E_1$$

$$\psi_2(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right)$$

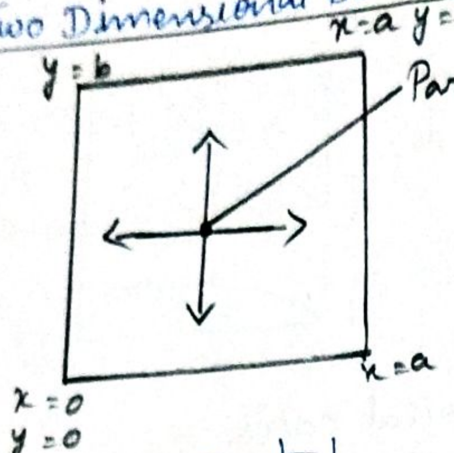
case (iii) For n = 3

$$E_3 = \frac{9h^2}{8ma^2} = 9E_1$$

$$\psi_3(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{3\pi x}{a}\right)$$

(4) Derive an expression for one dimensional Potential well extended for 2D and 3D Boxes.

i) Two Dimensional Boxes (2D)



Energy of the Particle.

$$E = E_{n_x n_y} = \frac{n_x^2 h^2}{8ma^2} + \frac{n_y^2 h^2}{8ma^2}$$

If $a=b$

$$E_{n_x n_y} = \frac{h^2}{8m}\left[\frac{n_x^2}{a^2} + \frac{n_y^2}{a^2}\right]$$

$$E_{n_x n_y} = \frac{h^2}{8ma^2}\left[n_x^2 + n_y^2\right]$$

The corresponding normalised wave function of the Particle in the two dimensional well is

$$\psi_{n_x n_y} = \sqrt{\frac{2}{a}} \sin\left(\frac{n_x \pi x}{a}\right) \times \sqrt{\frac{2}{b}} \sin\left(\frac{n_y \pi y}{b}\right)$$

$$\psi_{n_x n_y} = \sqrt{\frac{2}{a}}\sqrt{\frac{2}{b}} \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{b}\right)$$

$$\boxed{\therefore \psi_{n_x n_y} = \sqrt{\frac{4}{ab}} \sin\left(\frac{n_x \pi x}{a}\right) \sin \frac{n_y \pi y}{b}}$$

Example.

$n_x = 1 \quad n_y = 2$

$n_x^2 + n_y^2 = 1^2 + 2^2 = 1+4 = 5$

$n_x = 2, \; n_y = 1$

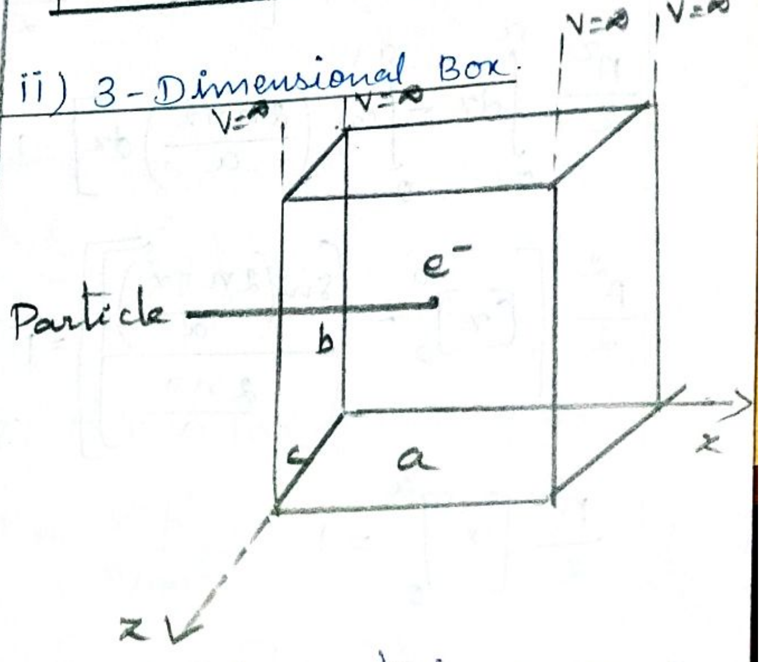
$n_x^2 + n_y^2 = 2^2 + 1^2 = 4+1 = 5$

$$E_{12} = E_{21} = \frac{5h^2}{8ma^2}$$

The corresponding wave functions is written as.

$$\boxed{\begin{aligned} \psi_{12} &= \sqrt{\frac{4}{ab}} \sin \frac{\pi x}{a} \sin \frac{2\pi y}{b} \\ \psi_{21} &= \sqrt{\frac{4}{ab}} \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \end{aligned}} \quad \text{— (4)}$$

ii) 3-Dimensional Box.



Energy of the Particle $= E_x + E_y + E_z$

$$E_{n_x n_y n_z} = \frac{n_x^2 h^2}{8ma^2} + \frac{n_y^2 h^2}{8mb^2} + \frac{n_z^2 h^2}{8mc^2}$$

If $a = b = c$

$$E_{n_x n_y n_z} = \frac{h^2}{8m}\left[\frac{n_x^2}{a^2}+\frac{n_y^2}{a^2}+\frac{n_z^2}{a^2}\right]$$

$$E_{n_x n_y n_z} = \frac{h^2}{8ma^2}\left[n_x^2+n_y^2+n_z^2\right] \quad \text{--- (1)}$$

The corresponding normalised wave function of the Particle in the three dimension well is written as

$$\psi_{n_x n_y n_z} = \sqrt{\frac{2}{a}}\sin\left(\frac{n_x \pi x}{a}\right)\cdot\sqrt{\frac{2}{b}}\sin\left(\frac{n_y \pi y}{b}\right)\cdot\sqrt{\frac{2}{c}}\sin\left(\frac{n_z \pi z}{c}\right)$$

$$\psi_{n_x n_y n_z} = \sqrt{\frac{2}{a}\times\frac{2}{b}\times\frac{2}{c}}\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{b}\right)\sin\left(\frac{n_z \pi z}{c}\right)$$

$$\boxed{\psi_{n_x n_y n_z} = \sqrt{\frac{8}{abc}}\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{b}\right)\sin\left(\frac{n_z \pi z}{c}\right)} \quad \text{--- (2)}$$

Example

$n_x = 1, n_y = 1, n_z = 2$

$n_x^2+n_y^2+n_z^2 = 1^2+1^2+2^2 = 1+1+4 = 6$

$n_x = 1, n_y = 2, n_z = 1$

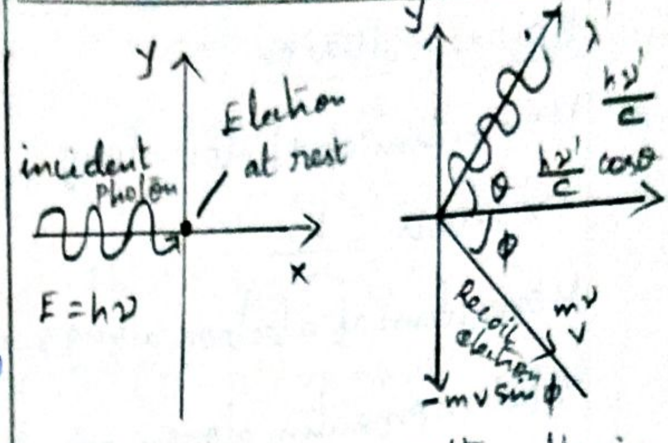
$n_x = 2, n_y = 1, n_z = 1$

$n_x^2+n_y^2+n_z^2 = 1^2+1^2+2^2$

$= 1+1+4$

$= 6$

Theory of compton Effect



a) Before collision b) After collision

Total Energy before collision

Energy of incident Photon $= h\nu$

Energy of electron at rest $= m_0c^2$

m_0 - rest mass of the electron

c - velocity of light

Total energy before collision

$= h\nu + m_0c^2$

Total Energy after collision

Energy of scattered photon $= h\nu'$

Energy of scattered electron $= mc^2$

m - mass of electron.

Total Energy after collision $= h\nu' + mc^2$

Applying law of conservation of energy

Total energy before collision = Total energy after collision

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$mc^2 = h\nu - h\nu' + m_0c^2$$

$$\boxed{mc^2 = h(\nu - \nu') + m_0c^2} \quad \text{--- (1)}$$

Total momentum along x-axis

Before collision

Momentum of Photon along x-axis $= \frac{h\nu}{c}$

Momentum of electron along x-axis $= 0$

Total momentum along x-axis $= \frac{h\nu}{c}$

After collision

Momentum of Photon along x-axis $= \frac{h\nu'}{c}\cos\theta$

Momentum of electron along x-axis $= m\nu\cos\phi$

Total momentum along x-axis after collision $= \frac{h\nu'}{c}\cos\theta + m\nu\cos\phi$

Applying the law of conservation of momentum

Total momentum before collision = Total momentum after collision

$$\boxed{\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + m\nu\cos\phi} \quad (2)$$

$$\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta = m\nu\cos\phi$$

$$\frac{h}{c}(\nu - \nu'\cos\theta) = m\nu\cos\phi$$

$$h(\nu - \nu'\cos\theta) = m\nu c\cos\phi$$

$$m\nu c\cos\phi = h(\nu - \nu'\cos\theta) \quad (3)$$

Total momentum along y-axis

Before collision

Momentum of Photon along y axis $= 0$

Momentum of electron along y axis $= 0$

Total momentum along y axis $= 0$

After collision

Momentum of Photon along y-axis

$$= \frac{h\nu'}{c}\sin\theta$$

Momentum of electron along y axis

$$= -m\nu\sin\phi$$

negative sign indicates -ve y direction

Total momentum along y-axis

$$= \frac{h\nu'}{c}\sin\theta - m\nu\sin\phi$$

Applying the law of conservation of momentum.

Total momentum before collision = Total momentum After collision

$$0 = \frac{h\nu'}{c}\sin\theta - m\nu\sin\phi$$

$$m\nu\sin\phi = \frac{h\nu'}{c}\sin\theta \quad (4)$$

$$m\nu c\sin\phi = h\nu'\sin\theta \quad (5)$$

squaring equ (3) and equ (4) and then adding

$$(m\nu c\cos\phi)^2 + (m\nu c\sin\phi)^2 = h^2(\nu - \nu'\cos\theta)^2 + (h\nu'\sin\theta)^2 \quad (6)$$

of equ(6)

$= m^2 \nu^2 c^2 \cos^2\phi + m^2 \nu^2 c^2 \sin^2\phi$

$= m^2 \nu^2 c^2 (\sin^2\phi + \cos^2\phi)$

$= m^2 \nu^2 c^2 \quad [\because \sin^2\phi + \cos^2\phi = 1]$

R.H.S of equ(6)

$= h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2\cos^2\theta) + h^2\nu'^2\sin^2\theta$

$= h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2\cos^2\theta + \nu'^2\sin^2\theta)$

$= h^2[\nu^2 - 2\nu\nu'\cos\theta + \nu'^2(\sin^2\theta + \cos^2\theta)]$

$= h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2)$

L.H.S = R.H.S of equ(6)

$$m^2\nu^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2) \quad \text{--- (7)}$$

Squaring equ(1) on both sides

$$(mc^2)^2 = (h(\nu-\nu') + m_0c^2)^2 \quad \text{--- (8)}$$

$m^2c^4 = h^2(\nu-\nu')^2 + m_0^2c^4 + 2h(\nu-\nu')m_0c^2$

$$m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4 \quad \text{--- (9)}$$

$\cancel{m^2c^4 = h^2(}$

Subtracting equ(7) from equ(9) we get

$m^2c^4 - m^2\nu^2c^2 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4 - h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2)$

$m^2c^2(c^2-\nu^2) = h^2\nu^2 - 2h^2\nu\nu' + h^2\nu'^2 + 2h(\nu-\nu')m_0c^2 + m_0^2c^4 - h^2\nu^2 + 2h^2\nu\nu'\cos\theta - h^2\nu'^2$

$m^2c^2(c^2-\nu^2) = -2h^2\nu\nu' + 2h(\nu-\nu')m_0c^2 + 2h^2\nu\nu'\cos\theta + m_0^2c^4$

$$\boxed{m^2c^2(c^2-\nu^2) = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4} \quad \text{--- (10)}$$

From the theory of relativity

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{\nu^2}{c^2}\right)}} \quad \text{--- (11)}$$

Squaring the equ(11) on both sides

$m^2 = \dfrac{m_0^2}{1 - \frac{\nu^2}{c^2}} = \dfrac{m_0^2}{\frac{c^2-\nu^2}{c^2}}$

$= \dfrac{m_0^2c^2}{c^2-\nu^2}$

$m^2(c^2-\nu^2) = m_0^2c^2$

Multiplying c^2 on both sides

$m^2c^2(c^2-\nu^2) = m_0^2c^2c^2$

$$m^2c^2(c^2-\nu^2) = m_0^2c^4 \quad \text{--- (12)}$$

Substituting equ(12) in equ(10)

$m_0^2c^4 = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$

$2h(\nu-\nu')m_0c^2 = 2h^2\nu\nu'(1-\cos\theta)$

(or)

$$\frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\frac{\nu}{\nu\nu'} - \frac{\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\boxed{\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)} \quad (13)$$

Multiplying c on both sides of equ (13)

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{hc}{m_0c^2}(1-\cos\theta)$$

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{hc}{m_0c^2}(1-\cos\theta)$$

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1-\cos\theta)$$

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$$

The change in wavelength is given by

$$\boxed{d\lambda = \frac{h}{m_0c}(1-\cos\theta)} \quad (14)$$

case (i)

$\theta = 0$

$$d\lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$d\lambda = \frac{h}{m_0c}(1-1)$$

$$= \frac{h}{m_0c} \times 0$$

$$\boxed{d\lambda = 0}$$

case 2:

$\theta = 90°$

$$d\lambda = \frac{h}{m_0c}(1-\cos 90°)$$

$$d\lambda = \frac{h}{m_0c}(1-0) \quad [\because \cos 90° = 0]$$

$$d\lambda = \frac{h}{m_0c}$$

Substituting for h, m_0 and c

$$d\lambda = \frac{6.625 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}}$$

$$\boxed{d\lambda = 0.0243 \text{Å}}$$

case - 3

$\theta = 180°$

$$d\lambda = \frac{h}{m_0c}(1-\cos 180°)$$

$$d\lambda = \frac{h}{m_0c}(1-(-1)) \quad [\because \cos 180° = -1]$$

$$d\lambda = \frac{h}{m_0c}(1+1)$$

$$d\lambda = \frac{2h}{m_0c}$$

$$d\lambda = 2 \times 0.0243 \text{Å}$$

$$\boxed{d\lambda = 0.0486 \text{Å}}$$

$\theta = 180°$

UNIT - V

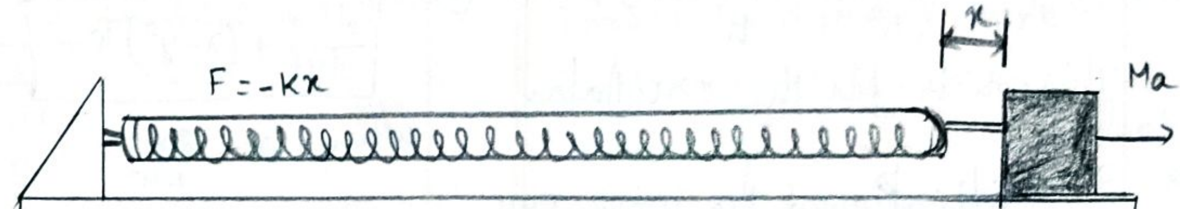
Applied Quantum Mechanics

The harmonic oscillator (qualitative) - Barrier Penetration and quantum tunneling (qualitative) - Tunneling microscope - Resonant diode - Finite Potential wells (qualitative) - Bloch's theorem for Particles in a Periodic Potential - Basis of Kroning-Penney model and origin of energy bands.

(1) Discuss about Harmonic oscillator (Qualitative)

Definition

A particle undergoing simple harmonic motion is called a harmonic oscillator.



Examples: Simple Pendulum, an object floating in a liquid.

If a applied force moves the Particle through x, then restoring force F is given by.

$$F \propto -x$$

$$F = -Kx \quad \text{—— (1)}$$

the Potential energy of the oscillator is

$$V = -\int F\,dx$$

$$V = K\int x\,dx = \frac{1}{2}Kx^2$$

$$V = \frac{1}{2}Kx^2 \quad \text{—— (2)}$$

K - force constant

In harmonic oscillator, angular frequency is given by.

$$\omega = \sqrt{K/m}$$

Squaring on both sides

$$\omega^2 = \left(\sqrt{K/m}\right)^2, \quad \omega^2 = K/m \quad K = m\omega^2$$

m - mass of the Particle

substituting K in equ (1)

$$V = \frac{1}{2}m\omega^2x^2 \quad \text{—— (3)}$$

wave equn for the oscillator

The time independent schrodinger wave equ for linear motion of a Particle along the x-axis

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad (4)$$

Substituting V in equ (4)

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - \tfrac{1}{2}m\omega^2x^2\right)\psi = 0 \quad (5)$$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2} - \frac{2m}{\hbar^2}\times\frac{1}{2}m\omega^2x^2\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \left(\frac{2mE}{\hbar^2} - \frac{m^2\omega^2}{\hbar^2}x^2\right)\psi = 0 \quad (6)$$

This equ is for the oscillator

Simplification of the wave equ

$$y = ax \quad (7)$$

$x = y/a$ where $a = \sqrt{\frac{m\omega}{\hbar}}$

$y = ax$
$dy = a\,dx$
$\frac{dy}{dx} = a$

$$\frac{d\psi}{dx} = \frac{d\psi}{dy}\frac{dy}{dx} = \frac{d\psi}{dy}\cdot a$$

Differentiating

$$\frac{d^2\psi}{dx^2} = \frac{d^2\psi}{dy^2}\cdot\frac{dy^2}{dx^2}$$

$$\frac{d^2\psi}{dx^2} = \frac{d^2\psi}{dy^2}a^2$$

$$\frac{d^2\psi}{dx^2} = a^2\frac{d^2\psi}{dy^2} \quad (8)$$

Substituting for $\frac{d^2\psi}{dx^2}$ and x^2 in equ (6)

$$a^2\frac{d^2\psi}{dy^2} + \left(\frac{2mE}{\hbar^2} - \frac{a^4y^2}{a^2}\right)\psi = 0$$

$$a^2\frac{d^2\psi}{dy^2} + \left(\frac{2mE}{\hbar^2} - a^2y^2\right)\psi = 0$$

$x = y/a$
$a = \sqrt{\frac{m\omega}{\hbar}}$
$a^2 = \frac{m\omega}{\hbar}$
$a^2 = \frac{m^2\omega^2}{\hbar^2}$
$a^4 = \frac{m^2\omega^2}{\hbar^2}$

Dividing through out by a^2

$$\frac{d^2\psi}{dy^2} + \left(\frac{2mE}{a^2\hbar^2} - y^2\right)\psi = 0 \quad (9)$$

Substituting for a^2

$$\frac{d^2\psi}{dy^2} + \left(\frac{2mE}{\frac{m\omega}{\hbar}\cdot\hbar^2} - y^2\right)\psi = 0 \quad (10)$$

(or)

$$\frac{d^2\psi}{dy^2} + \left(\frac{2E}{\hbar\omega} - y^2\right)\psi = 0$$

(or)

$$\boxed{\frac{d^2\psi}{dy^2} + (\lambda - y^2)\psi = 0} \quad (11)$$

where $\lambda = \frac{2E}{\hbar\omega}$

Eigen Values of the total energy E_n

The wave equ for the oscillator is Satisfied only for discrete values of total energies given by

$$\frac{2E}{\hbar\omega} = (2n+1) \quad \text{(or)}$$

$$E_n = \frac{1}{2}(2n+1)\hbar\omega$$

$$\boxed{E_n = \left(n+\frac{1}{2}\right)\hbar\omega} \quad (12)$$

$$E_n = \left(n+\frac{1}{2}\right)\frac{h}{\cancel{2\pi}}\cancel{2\pi}\nu$$

$$E_n = \left(n+\frac{1}{2}\right)h\nu \quad (13)$$

$n = 0, 1, 2, \ldots$

ν = frequency of the classical harmonic oscillator

$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}$

From equ (13)

i) Putting $n=0$ in equs (12) and (13)

$$E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}h\nu \quad \text{—— (14)}$$

This is called the ground state energy or the zero point vibration energy of the harmonic oscillator

$$E_n = (2n+1)E_0 \quad \text{—— (15)}$$

ii) The eigen values of the total energy depend only on one quantum number n.

<u>wave functions of the harmonic oscillator</u>

$$\lambda = \frac{2E}{\hbar\omega} = 2n+1$$

i) the normalisation constant N_n

$$N_n = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}(2^n n!)^{-1/2} \quad \text{—— (16)}$$

ii) exponential factor $e^{-y^2/2}$

iii) a Polynomial $H_n(y)$ called Hermite Polynomial in either odd or even Powers of y. The general formula for the n^{th} wave function

$$\psi_n = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}(2^n n!)^{-1/2} e^{-y^2/2} H_n(y)$$

—— (17)

Significance of zero point energy.

For lowest state $n=0$

$$E_0 = \frac{1}{2}h\nu.$$

In old quantum mechanics, the energy n^{th} level

$$E_n = nh\nu$$

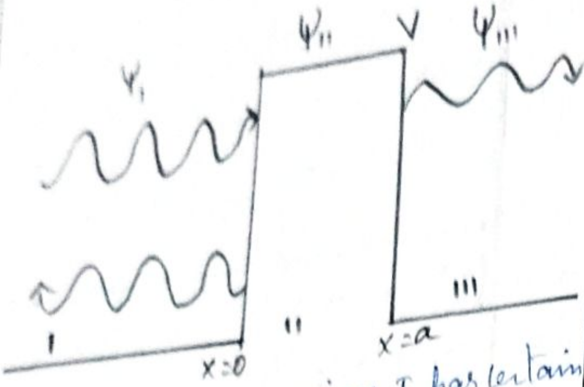
wave mechanics

$$E_n = (n+\tfrac{1}{2})h\nu.$$

(2) Discuss barrier Penetration and quantum Tunneling (out lab)

- According to classical ideas a particle striking a hard wall has no change of leaking through it
- The behaviour of a quantum Particle is different due to the wave nature associated with it.
- Electromagnetic wave strikes at the interface of two media, it is Partly reflected and Partly transmitted
- De-Broglie wave also Partly reflected from the boundary of the Potential well and Partly Penetrating through the barrier
- quantum mechanics leads to an entirely new result.
- It shows that there is a finite chance for the election to leak to the other side of the barrier
- The electron tunneled through the Potential barrier and hence in quantum mechanics, this phenomenon is called tunneling
- The transmission of electrons through the barrier is known as barrier Penetration.

Expression for transmission probability



- The Particle in region I has certain Probability of Passing through the barrier to reach region II and then emerge out on the other side in region III.
- The Particle lacks the energy to go over the top of the barrier, but tunnels through it.
- consider a beam of identical Particles all having Kinetic energy E.
- The beam is incident on the Potential barrier of height V and width a from region I.
- on both sides of the barrier V=0. This means that no forces act on Particles in regions I and III
- ψ_I represents the Particle moving towards the barrier from region I, while ψ_I represents the Particle reflects moving away from the barrier
- wave function ψ_{II} represents the Particle inside the barrier
- Some of the Particles end up in region III while the others return to region I.

T = Number of Particles transmitted / Number of Particles incident

This probability is approximately given by

$$T = T_0 e^{-2Ka}$$

where

$K = \frac{\sqrt{2m(V-E)}}{\hbar}$ and 'a' is the width of the barrier.

T_0 - constant close to unity.

- Probability of Particle Penetration through a potential barrier depends on the height and width of the barrier.

Significance of the study of barrier penetration problems.

- Tunneling is a very important Physical Phenomena which occurs in certain semiconductor diodes.
- The tunneling effect also occurs in the case of the alpha Particles
- The ability of electrons to tunnel through a Potential barrier is used in the Scanning Tunneling Microscope to study surfaces on an atomic scale of size.

what is the principle of scanning tunneling microscope
Explain the construction and working scanning tunneling microscope with a Suitable diagram

A scanning tunneling microscope or STM is a type of electron microscope. commonly used in fundamental and industrial research.

Principle

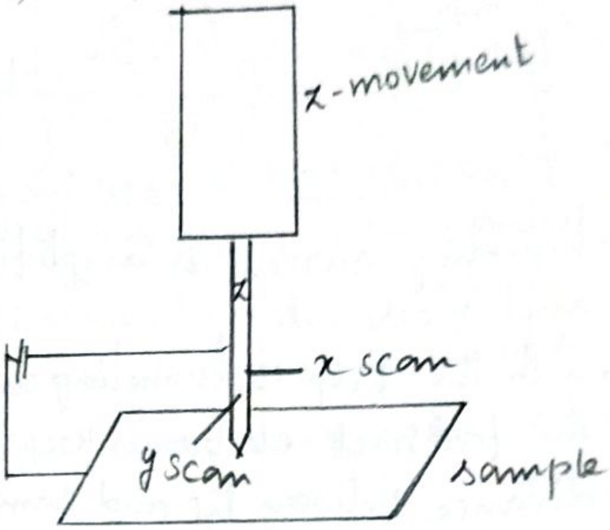
It is based on the concept of quantum mechanical tunneling of electrons.

- A sharp narrow conducting needle or tip is brought very near to the surface to be examined.
- A small voltage difference about 1V is applied between the tip and the surface of the material.
- This allows electrons to tunnel through the vaccum between them and results in tunneling current.
- Information about surface morphology is obtained by monitoring the tunneling current.
- The tips position scans across the surface and it is usually displayed in image form.

construction

components

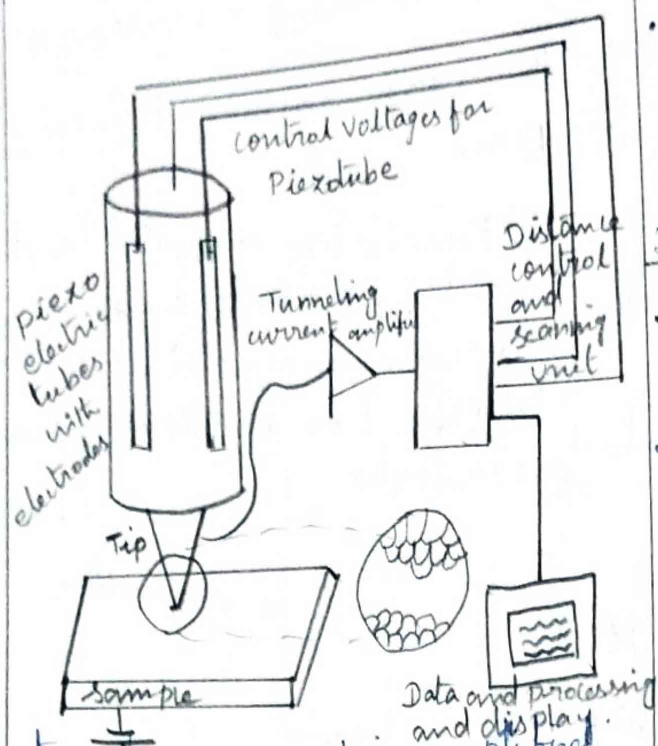
i) scanning needle tip.
ii) Piezoelectric controlled height and surface (x, y) scanner
iii) coarse sample to tip control.
iv) vibration isolation system
v) computer.



- Needle tip made of tungsten
- piezoelectric tube is provided
- Moving x, y, z directions
- coarse sample to tip control is used to bring the tip close to the sample.
- Any vibration
- Acquire data
- quantitative measurement.

working

- Bias voltage is applied between the sample and the tip
- when the needle is in positive potential, electrons can tunnel through the gap and setup a small tunneling current.



· tunneling current is amplified and measured.
· with the help of tunneling current, the feedback electronics keeps the distance between tip and sample constant.
· once tunneling is established sample can be verified and data are obtained.

Scanning

If the tip is moved across the sample in the x-y Plane, changes are mapped in images to present the surface morphology.
· The height z of the tip corresponding to a constant current be measured

Advantages of STM

· For an STM, good resolution is 0.1nm lateral resolution and 0.01nm depth resolution.
· To examine surfaces at an atomic level
· STMs are also versatile.
· used in ultra high vaccum water and other liquids and gasses.

Disadvantages of STM.

· STMs can be difficult to use effectively.
· A small vibration even a sound can disturb the tip and the sample together.
· Even a single dust particle can damage the needle

Applications of STM.

· It is a powerful tool used in many research fields and industries to obtain atomic sample imaging and magnification
· STM recently found manipulation of atoms.
· It is used to analyze the electronic structures of the active sites at catalyst surfaces.
· STM is used in the study of structure growth morphology electronic structure of surface, thin films and nano structures.

write a note on Resonant Diode

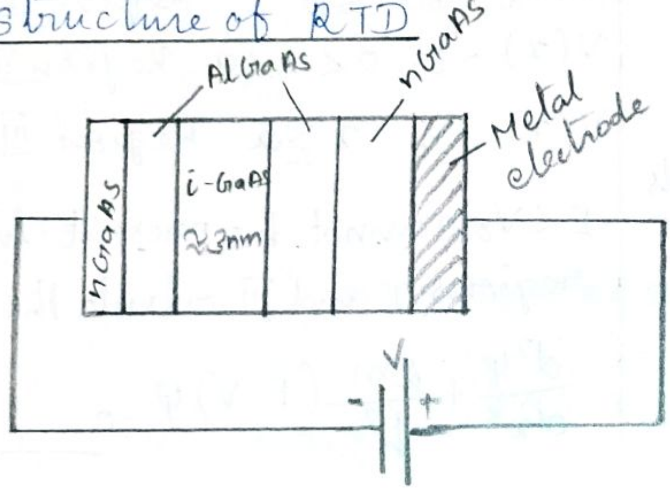
Definition

A resonant tunneling diode is a diode with resonant tunneling structure. The electrons can tunnel through some resonant states at certain energy levels.

Principle

when electron incident with energy equal to energy level of a Potential well of thin barrier, then the tunneling reaches its maximum value. This is known as resonant tunneling.

structure of RTD



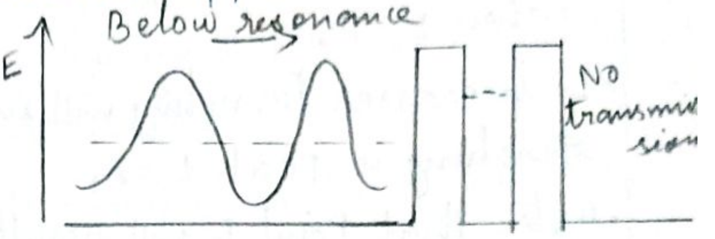
- structure is made by using n-type GaAs for the regions to the left and right of both barriers (regions 1 & 5)
- Tunneling is controlled by applying a bias voltage across the device.

Working

Tunneling control

By applying a bias voltage across the device.

without applied bias.

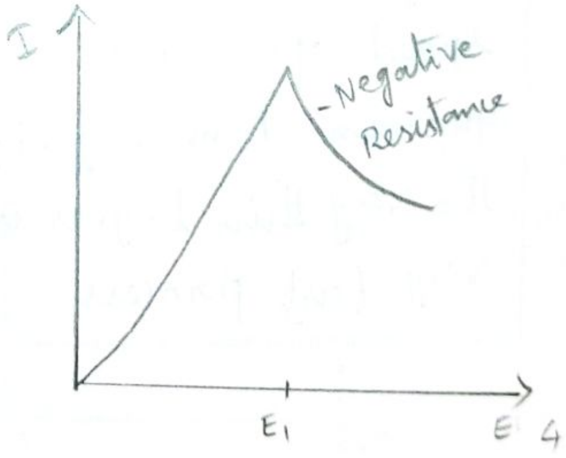


- No applied bias
- Very difficult to control the barrier height as well as the width of the Potential well to match the energy of the electron.

with applied bias

- when voltage is applied, the band diagram shifts.
- Voltage is Verified.
- Potential well matches with the energy of the electron wave.

Current-Energy characteristic for a resonant tunneling diode



- Incident electron energy E is very different state E_n.
- transmission is low.
- E tends to E_n, transmission will increase, becoming a maximum when $E = E_n$
- E increases tunneling will increase reaching a peak $E = E_1$
- After that point E will result a decreasing current.
- Decrease of current with an increase of bias is called negative resistance.

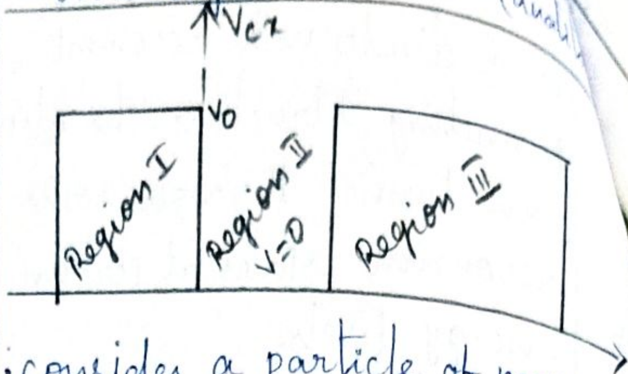
Application and uses of Resonant Tunneling Diodes

- Very good rectifier
- Used in digital logic circuits.
- Used in ~~ver~~ inverters, memory cells and transistors.

Advantages

- Very compact.
- They are capable of ultra high speed operations because the quantum tunneling effect through the very thin layers is a very fast process.

⑤ Particle in a finite potential well starting for schrodinger wave equ(ation)



- consider a particle of mass m
- x - direction between $x=0$ and $x=a$

Step - I

E - Total energy of Particle
V - Potential energy.

Potential energy is assumed to be zero with in the box.

$V(x) = V_0 \quad x \leq 0$ Region I
$V(x) = 0 \quad 0 < x < a$ Region II
$V(x) = V_0 \quad x \geq a$ Region - III

- $E < V_0$ cannot be present in regions I and III outside the box

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{——— (1)}$$

Step II

Three regions I, II, III separately, let $\psi_I, \psi_{II}, \psi_{III}$ be the wave function.

region I

$$\frac{d^2\psi_I}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi_I = 0 \quad \text{——— (2)}$$

for region II

$$\frac{d^2\psi_{II}}{dx^2} + \frac{2mE}{\hbar^2}\psi_{II} = 0 \quad \text{(3)}$$

for region - III

$$\frac{d^2\psi_{III}}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi_{III} = 0 \quad \text{(4)}$$

$$\frac{2mE}{\hbar^2} = k^2 \text{ and } \frac{2m(E-V_0)}{\hbar^2} = -k'^2 \quad \text{(5)}$$

(as $E < V_0$)

This equ in the three regions written as

$$\left.\begin{aligned} \frac{d^2\psi_I}{dx^2} - k'^2\psi_I &= 0 \\ \frac{d^2\psi_{II}}{dx^2} + k^2\psi_{II} &= 0 \\ \frac{d^2\psi_{III}}{dx^2} - k'^2\psi_{III} &= 0 \end{aligned}\right\} \quad \text{(6)}$$

step III

$\psi_I = Ae^{k'x} + Be^{-k'x}$ for $x<0$

$\psi_{II} = P.e^{ikx} + Q.e^{-ikx}$ for $0<x<a$

$\psi_{III} = Ce^{k'x} + De^{-k'x}$ for $x>a$

step IV

As $x \to \pm\infty$, ψ should not become infinite Hence $B=0$ and $C=0$

The wave functions in three regions

$\psi_I = Ae^{k'x}$

$\psi_{II} = P.e^{ikx} + Qe^{-ikx}$

$\psi_{III} = De^{-k'x}$

step V

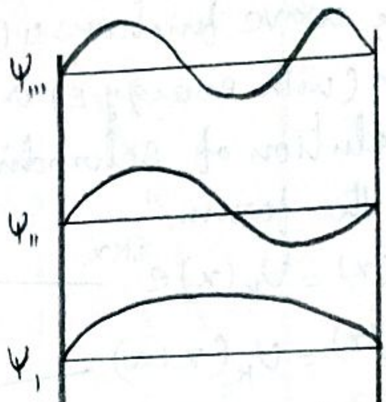
The wave function ψ and its derivative $\frac{d\psi}{dx}$ should be continuous in the region where ψ is defined.

$$\psi_I(0) = \psi_{II}(0)$$

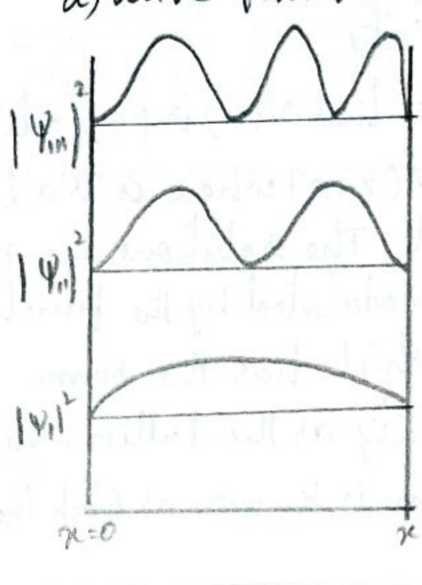
$$\left[\frac{d\psi_I}{dx}\right]_{x=0} = \left[\frac{d\psi_{II}}{dx}\right]_{x=0}$$

$$\psi_{II}(a) = \psi_{III}(a)$$

$$\left[\frac{d\psi_{II}}{dx}\right]_{x=a} = \left[\frac{d\psi_{III}}{dx}\right]_{x=a} \quad \text{(8)}$$



a) wave functions.



Probability densities inside non-rigid box

6. Explain Bloch's theorem for Particles in a Periodic Potential

Bloch theorem

It is a mathematical statement regarding the form of one electron wave function for a perfectly Periodic Potential.

Statement

If an electron in a linear lattice of lattice constant 'a' characterised by a Potential function $V(x) = V(x+a)$ satisfies schrodinger equ

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi(x) = 0 \quad (1)$$

then the wave functions $\psi(x)$ of electron (with energy E) is obtained as a solution of Schrodinger equ are of the form

$$\psi(x) = U_K(x)e^{ikx} \quad (2)$$

$$U_K(x) = U_K(x+a) \quad (3)$$

$U_K(x)$ is also periodic with lattice Periodicity.

The Potential $V(x)$ is periodic as $V(x) = V(x+a)$ where a is a lattice constant. The solutions are plane waves modulated by the function $U_K(x)$ which has the same Periodicity as the lattice. This thearem is known as Bloch theorem

Proof

we can write the property the Bloch functions equ (3)

$$\psi(x+a) = e^{ik(x+a)}U_K(x+a)$$

$$\psi(x+a) = e^{ikx}e^{ika}U_K(x+a)$$

since $\psi(x) = e^{ikx}U_K(x)$

$$\psi(x+a) = e^{ika}\psi(x) \quad (5)$$

(or)

$$\psi(x+a) = Q\psi(x) \quad (6)$$

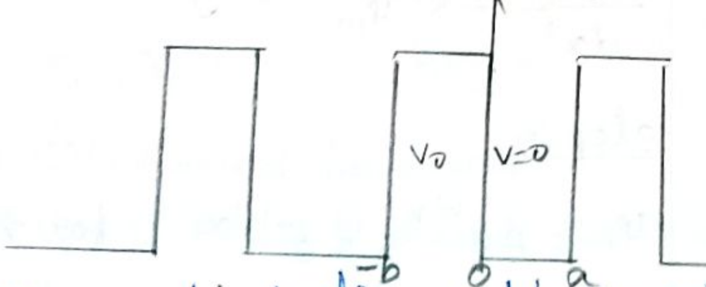
where

$$Q = e^{ika}$$

$\psi(x)$ is a single valued function

$\psi(x) = \psi(x+a)$. Thus Bloch theorem is proved.

7. Discuss of Kraning Penney model



- It was first discussed by Kronig and Penny in the year 1931.
- Behaviour of electronic Potential is studied by considering a Periodic rectangular well structure in one dimension.
- $0 < x < a$, the Potential energy is V_0.

The one dimensional schrodinger wave equ for two regions are written as

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-0)\psi = 0 \quad \text{(1)}$$

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \quad \text{(2)}$$

$$\alpha^2 = \frac{2mE}{\hbar^2}$$

and

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi = 0 \quad \text{(3)}$$

for $-b<x<0$

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \quad \text{(4)}$$

$$\beta^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

For both the regions the appropriate solution suggested by Bloch is of the form

$$\psi = e^{ikx}U_K(x) \quad \text{(5)}$$

Differentiating equ (5) and substituting in equ (2) and (4) and further solving it under boundary conditions.

$$\frac{P \sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka \quad \text{(6)}$$

where

$$\alpha = \frac{\sqrt{2mE}}{\hbar}, \quad P = \frac{mV_0 ba}{\hbar^2}$$

- The term P is called as scattering Power of the Potential barrier.
- It is a measure of strength with which the electrons are attracted by the Positive ions

From the graph $P \to 0$

$$\cos\alpha a = \cos Ka$$

$$\alpha = K \quad \alpha^2 = K^2$$

$$\frac{2mE}{\hbar^2} = K^2$$

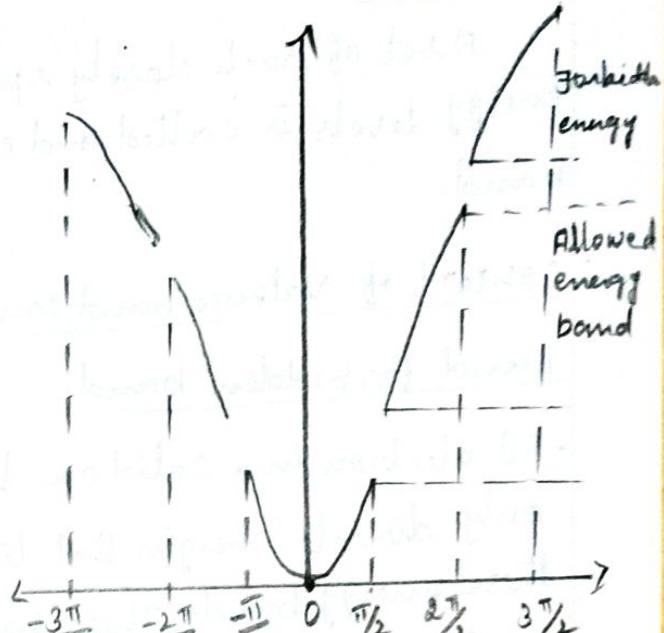
$$E = \frac{\hbar K^2}{2mE}$$

$$E = \frac{h^2K^2}{8\pi^2 m}$$

E - K curve.

The energy of the electron in the Periodic lattice.

$$E = \frac{h^2K^2}{8\pi m} \cdot K^2$$



7) Describe origin of energy bands in solid.

All the atoms of a solid, isolated from one another, can have completely identical electronic schemes of their energy levels.

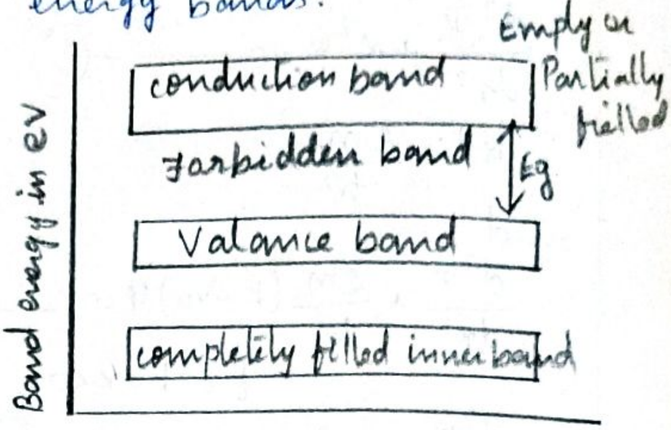
- Electrons fill the levels in each atom independently.
- closely spaced energy levels known as Permitted energy bands.
- Lower completely filled band is Valance band.
- Upper unfilled band is called conduction band.

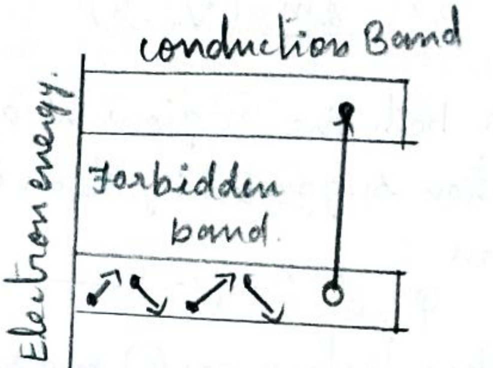
Definition:

A set of such closely spaced energy levels is called and energy band.

Concept of Valence band, conduction band forbidden band.

- All electron in a Solid can have only discrete energies that lie within these energy bands. These bands are called allowed energy bands.
- Band corresponding to valance electron is called Valence band.
- Band beyond forbidden band is called conduction band.
- Electrons in the outer most shell are called ~~conduction band~~ valance electron
- No allowed energy levels in some gaps called forbidden energy bands.





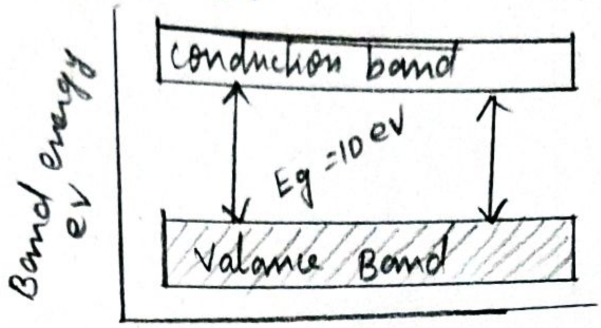
Classification of Metals, Semiconductors and insulators

Solids are classified into insulators, Semiconductors and conductors.

Insulators:

- Energy gap between conduction band and valance band is very high about 10 eV

forbidden energy band is very wide
conduction band is completely vacant and Valance band is completely filled.



<u>Semiconductors</u>

- Forbidden gap is very small
- Examples : Germanium and silicon
- Energy gap between conduction band and valance band is very small.
- 0.5 eV to 1eV
- conduction band is Partially filled and Valance band is Partially vacant.